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# Abstraction and Projection
## The Grounding of (at least some) Abstraction Principles

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ABSTRACT

José Zalabardo's recent book, *Pragmatist Semantics*, illuminates many important questions about the foundations of semantics. The main thesis is that the attribution of semantic properties is grounded on certain principles of abstraction defined over the real use of language. There are numerous problems concerning the introduction of ontological novelties through principles of abstraction. However, we will argue that the abstraction principles proposed by Zalabardo do not need to introduce any particularly suspect properties. Their basis could consist of projection processes in certain spaces so that some of the invariances originated can play, in quite a dynamical and open way, the role of the properties that are introduced for logical or semantic purposes. We will suggest the possibility of generalising our approach in relation to the application of abstraction principles in other areas.

KEYWORDS: *Semantics, Principles of Abstraction, Representationalism, Realism, Meaning as Use, Processes of Projection, Invariances.*

RESUMEN

El reciente libro de José Zalabardo, *Pragmatist Semantics*, ilumina muchas cuestiones importantes acerca de los fundamentos de la semántica. La tesis principal es que la atribución de propiedades semánticas se basa en ciertos principios de abstracción llevados a cabo sobre el uso real del lenguaje. Son numerosos los problemas que surgen a la hora de introducir novedades ontológicas en los principios de abstracción. Pero sostendremos que los principios de abstracción que propone Zalabardo no necesitan introducir ninguna propiedad particularmente sospechosa. Su base podría consistir en procesos de proyección en determinados espacios de manera que algunas de las invarianzas originadas desempeñen, de una manera muy dinámica y abierta, el papel de las propiedades que quieren introducirse para fines lógicos o semánticos. Sugeriremos la posibilidad de generalizar nuestro enfoque en relación con el empleo de principios de abstracción en otros ámbitos.

PALABRAS CLAVE: *semántica, principios de abstracción, representacionalismo, realismo, significado como uso, procesos de proyección, invarianzas.*

> "[…] one can often reduce one's ontological commitments by expanding one's logic."
>
> H. Field, *Science without numbers*, 2016. (second edition), preface first edition: p. iii.

In a recent book, José Zalabardo appeals to certain abstraction principles as a bridge to reconcile two positions.[1] One of these positions is the realistic interpretation of what we think and express through declarative sentences incorporating predicates such as "being morally right," "believing something," "desiring something," "having a certain meaning," or "being true." Such realism is the natural attitude we commonly adopt towards the predicates involved in those sentences. They always seem to represent reality as "being a certain way." The other position is a way of understanding meaning according to which its ultimate foundation must be the way we actually use language. This appeal to the use of language as the foundation of meaning, and consequently also of reference, is again a reasonable philosophical approach. This is especially true in light of the problems associated with assuming that, at least in declarative sentences, the foundation of meaning consists in relationships independent of language use between the referential terms involved in the sentences and certain entities in reality. Compatibility between the two aforementioned positions would be desirable. However, can this compatibility be adequately established through abstraction principles?

The abstraction principles proposed by Zalabardo assume numerous simplifications and idealisations, many of them normative. The principles require the existence of some equivalence relations between language uses. But the identification of these equivalence relations depends on multiple decisions and commitments. The formulation of abstraction principles through conditions that are not only sufficient but also necessary is again the result of simplifications and idealisations. Also, it is an idealisation that an abstraction function assigns properties as references of the analysed predicates. All these simplifications and idealisations should be justified by something more fundamental. The possibility we will explore is that the source of such justification consists in projection processes generating certain invariances, playing some of these invariances the role attributed to the properties introduced through abstraction principles.

The paper is structured as follows. We will begin by discussing Zalabardo's use of abstraction principles. Next, we will explain our approach. Finally, we will connect our proposal with that of Zalabardo and make some general considerations.

## I. Abstraction Principles in the Pragmatist Semantics of Zalabardo

The abstract unifies reality by making it intelligible. Both mathematical and logical entities, as well as universals, and properties in general, are paradigmatic examples of abstract entities.[2] Traditionally, it is assumed that the abstract is somehow also present in concrete things, and that we can access it by disregarding particular details, that is, by simplifying and idealizing. Often, these two procedures coincide. In any case, the function of abstraction would be to obtain the abstract within concrete things through certain simplifications and idealisations.

However, the nature of abstraction processes has never been clear. How can we arrive at something abstract through simplifying or idealising? How can we reach something that seems to exist neither in space nor in time and that cannot have causal relationships with us? Frege faced these problems when seeking a logical foundation for mathematics. He made a highly original proposal. If we are capable of relating entities $x$ and $y$ through a certain equivalence relation $R$, then we can use that equivalence relation to define an abstraction function $f$ that asserts identities between certain abstract entities that are its values. Thus, abstraction principles can be formulated with the following structure:

$$(APs)\ \forall x \forall y\ (f(x)=f(y)\ \textit{iff}\ Rxy)$$

The function $f$ has as its domain those entities, whether linguistic or mental, that we can say refer to a certain abstract entity. And its values are certain abstract entities. In turn, $R$ is an equivalence relation, a binary relation internal to the domain of $f$ with reflexive, symmetric, and transitive properties. *APs* assert that $f$ assigns the same abstract entity as the value of $x$ and $y$ if and only if a certain equivalence relation $R$ holds between $x$ and $y$, that is, if $x$ and $y$ belong to the same equivalence class according to $R$.

Frege employed abstraction principles to introduce numbers based on the more basic equivalence relation of equinumerosity between sets.

More recently, abstraction principles have been used by other authors in the same vein.[3] A simple presentation is as follows:

> *(APs-num)* $\forall x \forall y$ *( The number of the* $x$ = *The number of the* $y$ *iff the sets* $x$ *and* $y$ *are equinumerous )*

*APs-num* offer a partial contextual definition of what numbers are. They tells us when there are the same numbers of things in any two sets without fully explaining what numbers themselves are.

In general, the function $f$ assigns the same abstract entity to entities $x$ and $y$ if and only if there exists a certain equivalence relation $R$ that places $x$ and $y$ in the same class. However, *APs* do not precisely identify the nature of the abstract entities that are the values of $f$. They only establish a necessary and sufficient condition for the correct assignment of the same abstract entity to the elements of its domain. So, it seems desirable to have other means to specify the abstract entities to be defined, means that are independent of the *APs* themselves. And this situation is a source of long-standing problems and discussions.[4]

Zalabardo uses *APs* to attribute realistic representational references to problematic predicates such as "being morally correct," "believing something," "desiring something," "having a certain meaning," or "being true." These predicates describe certain peculiar ways of being. It is assumed that their references are certain properties, and *APs* allow the introduction of these properties as identities among the values of an abstraction function $f$ under certain conditions. Different conditions are proposed for each type of problematic predicate, and they are reached through a discussion of the alternatives at play when explaining how the actual use of such predicates is normatively regulated.

Finding those conditions is not easy. Moreover, any assignment of specific properties faces "open-question arguments," undermining the performance of the representational function. In response to this, the conclusion is usually deflationist: the problematic predicates do not have a representational function. However, Zalabardo proposes another possibility: that the meaning of those predicates can be representational without the basis for that meaning being representational. The basis can be pragmatist. Moreover, we do not need to assign completely specific properties. It is sufficient to employ certain *APs* allowing us to assert that the predicates must have the same reference under certain conditions of use. The properties capable of being the references of predicates with a meaning grounded in their use can be identified through certain

*APs* based on "conditions of synonymy" settled by their procedures of ascription.

The domain of quantification in these *APs* are certain predicates with an intuitively realistic but philosophically problematic representational content. The *APs* used would have the structure presented above. The abstraction function *f* would always assign properties. And the necessary and sufficient conditions for such assignment would be established by the existence of synonymies generated by the actual procedures of ascription of the problematic predicates. That appeal to use leads Zalabardo to consider his approach akin to pragmatism and also to the ideas of Wittgenstein in the *Philosophical Investigations*.

The basis of the meaning of a sentence would always consist in the procedure actually employed by speakers to regulate its acceptance. In the case of ascriptions of beliefs and desires, for example, Zalabardo assumes the procedures of ascription described by Daniel Dennett in his analyses of the intentional stance. However, he modifies them by dispensing with Dennett's recourse to the beliefs and desires the agent "ought to have," giving greater weight to their role in "predicting behaviour". In the case of attributions of meaning and truth, Zalabardo assumes Quine and Davidson's strategy of radical interpretation. Meaning and truth are attributed according to balances between such attribution and the ascription of beliefs based on the acceptance of statements. But there are again some changes. The balances would be guided by a principle of charity "understood in a certain nuanced way" and "supplemented" by non-linguistic evidence. All the aforementioned nuances, modifications, and supplements are simplifications and idealisations that allow for the delineation of equivalence relations capable of featuring, on the right side of the corresponding *APs*, conditions for asserting the left side of such principles.[5]

The proposed *APs* lack independent characterisations of the involved properties. However, Zalabardo argues, such characterisations are unnecessary. A right-to-left reading of the *APs* would suffice to ensure the assignment of references to those predicates. Nonetheless, we must insist that this assignment will inevitably be partial and non-specific. The properties that appear as values of the abstraction functions are determined in a very incomplete manner.

Zalabardo contends that when there are independent characterisations of references, as is the case with predicates thematised by science, the ascription of references is also regulated by the normativity present in certain uses of language. This allows for a generalisation of his prag-

matist approach. Essentially, Zalabardo maintains that use determines all the meanings expressed in our languages, and therefore all the involved referential functions. This generalisation also extends to the predicates present in the metasemantic statements used to express the *APs* themselves. In other words, the pragmatist semantics would apply to itself. The pragmatist semantic descriptions would have a pragmatist semantics.

In these generalisations, Zalabardo's approach shifts. There is an appeal to a more fundamental notion of language use. We might think that abstraction principles still exist, allowing for the introduction of properties as realistic references based on the use. However, there would be here something we could call "background abstraction." This background abstraction is proposed as the ultimate foundation of the *APs*.

Is that way of grounding meaning adequate? There is a noticeable contrast between, on one hand, the tentative nature with which the different conditions that should appear on the right side of the *APs* are discussed and proposed, and on the other hand, the firmness with which the *APs* assert the existence of certain properties as referential semantic values that are identical if and only if the previous conditions are met. Moreover, even if this "if and only if" is weakened, and only sufficient conditions are asserted, what is established on the right side of the *APs* must be truly sufficient to assert the left side of the APs. But given that these conditions are formulated so tentatively, it becomes very difficult to guarantee that.

Zalabardo addresses this contrast under the label "*The Harmony Problem*." This problem concerns ensuring that non-representational conditions of semantic grounding, such as those offered on the right side of the *APs*, can indeed be at least sufficient conditions to assert the left side of such *APs*. In the following section, we will present a general strategy to tackle this problem.

It is also surprising that *APs* are used as part of a pragmatist claim regarding the use of language as the ultimate foundation of semantics, in a sense very close to Wittgenstein. The search for ultimate foundations was always far from the philosophical horizon of pragmatists and of the Wittgenstein to whom reference is made. Presumably, it would also be outside their philosophical horizon to assume abstract entities such as those obtained through the application of *APs*. Pragmatists and Wittgenstein were very opposed to the kind of simplifications and idealisations necessary to obtain the *R* relations appropriate to formulate the *APs* formulated by Zalabardo. The use of language does not seem to be the sort of phenomenon that is so regulated as to categorically support those

*APs*. In any case, if some *APs* are proposed for some aim, their functioning should be much more flexible and open than suggested by the assertion of the equivalence relations *R*, and by the connection between the left and right sides of such principles through necessary and sufficient conditions.

## II. Processes of Projection in the Basis of Abstraction Principles

How can a family of statements have a representational function without the predicates involved being connected to entities in reality with independence of the the use of language? Zalabardo claims that this can be achieved through certain *APs* where the use of these predicates generates equivalence classes capable of guaranteeing the assertion of their right-hand side. This would solve *The Harmony Problem*. However, all this can be accepted without the referential values of *f* being properties. There is no necessity for that. Not even assuming that the representational role of the statements involves representing reality as "being a certain way." As we will show, it is possible to represent reality as "being a certain way" without the referential values of the abstraction function *f* being properties.

We will suggest a grounding of the *APs* based on projection processes. The idea is quite simple. Let's say a phenomenon projects onto another phenomenon when the former can acquire certain particular values with respect to the latter. Some combinations of phenomena form complex projection spaces in which other phenomena are projected. Our mind, for example, does that continuously. And all the reality we can access thus becomes a series of projection values structured in very diverse ways, allowing for the identification of some invariances. We will suggest that the *APs* can be grounded in projection processes.[6]

Our proposal will be very dynamic and open. The relevant projection processes take place in our experience and thought, whether conscious or unconscious. They do not strictly generate equivalence relations or equivalence classes. However, by allowing the identification of invariances, they produce something similar: quasi-equivalence relations, which we will call "invariance-relations," and quasi-equivalence classes, which we will call "invariance-classes." The basis for two predicates to refer to the same property, in the context of an *AP,* will be that there exists a certain invariance-class in the projections of reality in our experience or thought, enabling the use of both predicates in the same way.

Let us start with the observation that the *APs* themselves are abstract entities. Could we thus use certain principles *AP1, PA2, ..., PAn* to ground another *PAi*? This could be feasible if the principles *PA1, PA2, ..., PAn* were well-defined and clearly different from *PAi*. However, this is not the case with the various *PAs* used by Zalabardo. These *PAs* are not grounded in other clearly different and well-defined *PA*. Can we find such a foundation in what we previously referred to as "background abstraction"? We cannot. It is not defined with sufficient precision. Therefore, we cannot even determine if it is sufficiently different from the *APs* that should be grounded by it. In particular, we do not know how it is related with the sorts of considerations that allow the definition of equivalence relations *R* in the semantic attributions of meaning and truth, in the attributions of propositional attitudes, etc.

We will propose another way to ground *APs*. It is different from the previous approaches. It consists of offering a foundation based on projection strategies. We will explore this path by making a concrete proposal that deviates from the usual philosophical options. The central hypothesis is that abstraction could be based on more fundamental processes of projecting phenomena into certain multidimensional spaces. We project phenomena into various spaces of diverse dimensions, seeking to identify relevant invariances, and these invariances determine the references of the predicates we use to describe the phenomena we project.

Our model will be the mathematical concept of a "tensor" as used in geometry and topology, but not many details will be necessary. We can project any phenomenon onto an n-dimensional vector space. The space is generated by a certain vector basis. What we obtain from the projection is a set of projection values. We can have several kinds of values. The values may be completely independent of the vector space of projection, in which case they would take the form of "scalars," which are tensors of order zero. We can have "vectors," which are tensors of order 1. Vectors have a projection value in each dimension of the space. We can also have "matrices," which are tensors of order 2. An $n \times m$ matrix has $n$ rows and $m$ columns. Each row is a vector, and each column consists of various values in each dimension of the space. Finally, we can have "matrices of matrices," which are tensors of order higher than 2, composed of matrices "nested" within other matrices.

In a projection, there is dimensional complexity derived from the number of dimensions necessary to adequately project the phenomena. And there is also a highly important structural complexity generated by the tensorial order. Some phenomena can be adequately represented by

tensors of order 0 or 1, but others require tensors of order 2 or higher. For example, the pressure exerted on a body across its entire surface, or how it might deform or explode, are phenomena that can only be adequately represented with tensors of order 2 or higher.

A very important feature of tensors is that they are not simply identified with particular projection values. Although they have specific projection values, they are invariant with respect to many of their projections. We can have the same tensors (whether scalars, vectors, matrices, etc.) across various changes in the projection spaces. The most common changes are rotations, translations, and scale changes in the vector bases generating the projection spaces. Similarly, certain deformations in the projection spaces can be considered. As we will see shortly, other changes can also be considered. The main point is that projections can be very different without necessarily changing what is being projected.

The invariances obtained are only detectable through the projections. However, they are not dependent on the particular projections. This offers a very interesting kind of "objectivity". And it is tempting to use the notion of invariance to analyse other notions related to objectivity, such as truth, justification, rationality, or even reality. We will not do this here.[7] What we will propose is a close link between the notion of invariance and the *APs*.

As we have mentioned, we want to take the concept of a tensor as our model. Based on this model, we will propose two enhancements. First, let us admit projection spaces composed of dimensions not only quantitative but also qualitative, formed by all sorts of classificatory and comparative concepts.

Before introducing the second enhancement, we need to elaborate on the notion of invariance. We can understand it through the notion of invariance-relations *Ri*, which can be defined as follows:

> Invariance-relation *Ri* = It is a relation between projection values that is maintained under certain changes or transformations in the projection space.

*Ri* relations remain constant across different projections, either within the same projection space or across different projection spaces. They can be considered as "patterns of projective behaviour". They are very dynamic and relative. The invariance-relations with respect to certain changes in space may not be invariant with respect to other changes. They are always relative to certain kinds of changes in the projection space.

The aim of our second enhancement is to highlight invariances that consist in maintaining certain relevant relations between the projection values under other conditions such as 1) changes in the projection dimensions, 2) changes in the values of vectors, i.e. tensors of order 1, or in the rows and columns of matrices, i.e., tensors of order 2 or higher, and 3) changes in the "nesting structures" in tensors of order higher than 2.

The existence of invariances allows for the identification of important patterns in the projected phenomena. Sometimes these patterns are quite stable; other times they are more unstable. In any case, they form part of "objective reality". And we must remember that, in our case, they will always be invariances concerning the use of certain problematic predicates. We would detect these invariances by projecting the use of those predicates into certain multidimensional spaces. And the important features will always be the invariance-relations *Ri* that may arise among the projection values.

Now, let us introduce the notion of "relevant invariance":

> Relevant invariance = An invariance-relation *Ri* obtained in all the mentioned conditions allowing a better control, prediction, explanation, or understanding of the projected phenomena.

Many invariances are irrelevant. What makes them relevant is their contribution to controlling, predicting, explaining, or understanding the phenomena. Along with translations, rotations, and scale changes in the projection spaces, and along with other transformations of the projection spaces, the new conditions cover all possibilities in which some *Ri* relations can remain constant while changing the projection values.

By being able to consider qualitative dimensions, the relevant invariances may not have an exclusively mathematical meaning. However, their meaning will always be strongly empirical. They will allow for recognizing the existence of "objective patterns" in reality through the accessible data or information.

In themselves, the *Ri* relations are not equivalence relations. They can be formed by any set of tuples, each consisting of a series of projection values. However, an *Ri* relation could be so close to being an equivalence relation established among projection values as to be considered "practically equivalent". To better explain this, let us introduce the concept of "invariance-class":

> Invariance-class $\hat{}Ri[x]$ = The entities related to x according to a certain invariance-relation *Ri*.

Nothing prevents the invariance-classes from being equivalence classes. Moreover, we can consider that there is an equivalence class $[x]$ generated by a certain equivalence relation *R*, defined in the domain to which $x$ belongs, when there is a invariance-class $\hat{}Ri[x]$ generated by a certain relevant invariance-relation *Ri*, obtained among the projection values of elements of that domain.

Our goal is to provide a response to *The Harmony Problem* by offering plausible conditions to guarantee the assertion of the left-hand side of some *APs* slightly different from those proposed by Zalabardo. On the left-hand side of these *APs*, there would not strictly be properties as referential values of the abstraction function *f*, but invariance-classes. Introducing properties will play a "pleonastic" role. This role can be very important in semantics or logic. Attributing properties as the semantic values of predicates is much simpler and easier than attributing invariance-classes in relation to projection processes carried out in some projection spaces. However, ontologically it will only be a pleonasm. And it will also be a pleonasm in metasemantics. Thus, in our ontology and metasemantics, we avoid what Zalabardo calls "*The RR Assumption*": the claim that statements acquire their representational function by connecting the involved predicates with the properties of reality attributed as their referential values.

Now, we have all the ingredients to ground *APs*, especially those used by Zalabardo. Consider the following structure we can call "Projection Principles":

$$(PPs)\ \forall x \forall y,\ldots,\forall z\ (I(x)=I(y)=\ldots I(z) \text{ iff } Ri(p(x), p(y),\ldots p(z))$$

The values of the function *I*, which can be called "the invariance function," are invariance-classes. *PPs* assign the same invariance-classes to elements of a projection domain $D=\{x, y,\ldots z\}$ according to a relevant invariance-relation *Ri* established among the projection values $p(x), p(y),\ldots p(z)$. These projection values can be obtained in the same projection space or in different projection spaces, according to the conditions we have established.

*Ri* is a relation between projection values. In the case of the problematic predicates addressed by Zalabardo, the dimensions of the projection spaces would be the factors of the use mentioned in his analyses. In

fact, *Ri* can be very heterogeneous. But at the end of the day, it is simply a set of tuples of projection values. We have seen that *Ri* allows for the formation of relevant invariance-classes. The class of tuples involving *x* with respect to *Ri* will be the invariance-class ^*Ri[x]*. And each different *Ri* can give place to different invariance-classes.

What a particular *PP* would assert is that the existence of a relevant invariance-relation *Ri* is a necessary and sufficient condition for assigning the same invariance-class to the elements of a projection domain *D* according to *Ri*. And the crucial point is that even though *Ri* does not strictly define an equivalence relation in the domain, it can perfectly behave as an equivalence relation in some circumstances.

We emphasize that invariance-classes are not properly equivalence classes. Their existence depends on the open series of projections we carry out. We can say that they are "quasi-classes" based on "quasi-equivalence relations."[8] The ontological commitment of abstraction would be based on the objective existence of these quasi-classes. Abstraction can be understood as affirming that these invariance-classes exist and that some of the projected phenomena belong to them. In other words, what underpins the existence of the equivalence classes mentioned in the *APs* does not need to be an equivalence class. It suffices that it be an invariance-class.

There is an obvious parallelism between the structure of *APs* and the structure of *PPs*. And this parallelism is aimed because we want to argue that:

1. *PPs* are much more basic and natural than *APs*.
2. *APs* can be grounded in *PPs*, in the sense that they can be practically obtained from them.
3. *PPs* can solve some of the problems left unresolved by *APs*.

We will leave points 2 and 3 for the next section and will now address very briefly point 1.

A projection process is a process in which a phenomenon is projected onto certain projection spaces, seeking some relevant invariances. These relevant invariances are important for guiding the behaviour, both epistemic and non-epistemic, of the system carrying out the projection processes. And this is also what we do when we inquire about the meaning, or more specifically the references, of the predicates we use when using language to describe aspects of reality.

All the concepts we are employing are sufficiently general to affirm that reality is full of projection processes, carried out by entities like us, or by many other sorts of entities. Projection processes are extremely basic and natural. Any sensitivity to certain information or data is a projection process. However, some entities are capable of carrying out highly sophisticated projection processes. The projection processes that can be reconstructed as applying *APs* would be of that kind.

In sum, the sort of "necessitation" or "guarantee" we need in order to solve *The Harmony Problem* has to come from the way we are connected with the rest of reality. And the notion of projection, in the sense we have explained, offers a very suggesting way to analyse that connection.

## IV. More About Principles of Projection

We have not exactly "derived logically" the *APs* proposed by Zalabardo. However, we have offered a suggestive way to underpin principles very similar to those he proposes. Asserting *PPs* can justify the assertion of *APs*. And they can do so without resorting to *The RR Assumption*. Representation through projections would not entail it. The meanings of statements representing reality would derive their meanings through complex projections, with the meanings attributed to the parts of those statements simply being some more or less stable invariances obtained in those projection processes.

Our results allow for a realistic interpretation of the problematic predicates analysed. They do so by providing more ontological room for the functioning of *APs* strongly depending on simplifications and idealisations. Furthermore, perhaps in a manner more consistent with the role of real language use in both pragmatism and Wittgenstein's philosophy.

The representational role of the problematic predicates is preserved. What we have offered reconceptualises, or at least complements, Zalabardo's proposal. Quasi-relations and quasi-classes are generated through projection processes aimed at representing phenomena. And projecting, in the sense we are using this notion, is a highly basic way of representing that does not require *The RR Assumption*.[9]

Projection processes generate something very close to the equivalence classes required in abstraction principles. In some contexts, this would be close enough. That way, the *APs* formulated by Zalabardo would be acceptable in semantics and logic, and could play an important role, even a cognitive one, in those domains. We have argued that they

should be considered "pleonasms" when doing ontology and metasemantics. However, this sort of contextualisation does not necessarily relativize the notion of existence. The only thing that is relativized are our ways of conceptualising what exists and our ways of talking about it.

Distinguishing domains of application is crucial. Let's consider the distinction between "internal" and "external" existence claims within a conceptual framework, in the sense of Carnap. A conception of *APs* that does not distinguish domains of application would allow for identity assertions between entities existing externally to any conceptual framework in virtue of some generalised equivalences among statements internal to those conceptual frameworks. Or, let us consider the distinction between possessing "phenomenal concepts" and the actual existence of "phenomenal properties". A conception of *APs* that does not distinguish domains of application would permit identity assertions between phenomenal properties existing in reality in virtue of some generalised equivalences in the use of phenomenal concepts. Surely, this is not what we would desire.

An important consequence of our approach is that it calls into question "indispensability arguments" for the existence of abstract entities. It may seem practically indispensable to talk about properties and propositions in semantics and logic, just as in mathematics it may seem practically indispensable to talk about numbers, sets (or classes), Hilbert spaces, etc. However, this does not compel us to assume their existence. Ontologically and metasemantically, there can be reasons to understand all those entities in a "pleonastic" way.[10]

Among the paradigmatic examples of abstract entities are also geometric figures. Our approach will apply very well to them. We project "concrete figures" in certain spaces. And we get certain quasi-relations and quasi-classes. Thinking that there exist Lines, Triangles, Circles, Tori, etc., as abstract entities to which some expressions refer, and thinking that the very projection spaces exist as abstract entities, is fully justified in mathematical contexts. But in ontology and metasemantics it is not.

In summary, we have proposed a reappraisal of *APs* based on the invariances arising in certain projection processes. Abstraction is understood as the result of projection processes capable of generating invariance-relations and invariance-classes that may eventually satisfy the conditions of an equivalence relation defining certain equivalence classes. *APs* would be a special way of presenting that result. And although assigning properties as values of the abstraction function *f* may play an im-

portant role in semantics and logic, it does not need to play any such role in ontology or metasemantics.

To what extent can our approach to abstraction be generalised? Can it also encompass *APs* allowing references to other mathematical and logical entities? Indeed, every purported abstract entity, starting with properties and equivalence classes themselves, could have the nature of an invariance-class in the sense that has been introduced. Furthermore, it is even plausible to affirm that rational intuition and *a priori* reasoning develop through projections in the proposed sense. But we cannot address this issue here in full detail, particularly whether such an approach can resist objections commonly raised against psychologism, or against any form of naturalism.

However, one thing worth of consideration is that the "modal scope" of invariances regarding changes in projection processes is maximal. If all our accesses to reality are projective, then nothing could exclude that maximal modal scope apart from what the processes of projection themselves might offer.

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NOTES

[1] Zalabardo (2023).

[2] The notion of "abstract entity" cannot be defined through necessary and sufficient conditions. See Falguera, Martínez-Vidal, and Rosen (2022). Something analogous occurs with the notion of "concrete entity." The characterisation must follow other paths.

[3] The most prominent representatives of that neo-Fregeanism are Bob Hale and Crispin Wright. We will not delve into the details of the approach.

[4] Suppose we use *PAs-num* assigning values to *f* in the following way: instead of assigning *0*, *f(x)* and *f(y)* assign *0.000000...1*; instead of assigning *1*, *f(x)* and *f(y)* assign *0.000000…9*; instead of assign *2*, *f(x)* and *f(y)* assign *1.000000...9*, and so on —insert the appropriate number of zeros so that the argument holds strength. The possibility of assigning such non-standard values to *f* endangers our ability to understand numbers "exclusively" through *PAs-num*.

[5] For the ascription of propositional attitudes, see Zalabardo (2023), chap. 6. For meaning and truth, see Zalabardo (2023), chap. 7.

[6] "Projecting" has various meanings: 1) to consider as real something that might only be in our mind, 2) to make predictions, 3) to create and test some designs, 4) to acquire certain values in the dimensions of a given space. The primary sense for us will be 4, a geometric and topological sense. However, we will leave open the possibility suggested by sense 1, which is common in ethics and psychology.

[7] Nozick (2001) explores some of those topics.

[8] In a sense very close, but with differences in how the notion of projection is understood, to the sense analysed by Blackburn (1993). See note 6.

[9] Furthermore, even if Russell's contextual elimination of classes were ultimately correct, the representational role of the quasi-classes obtained through the quasi-relations of invariance would not be compromised. And the relevant invariances obtained could have the highest degree of objectivity. For a recent defence of Russell's position, see Klement (2010).

[10] For a related position, see Schiffer (2003).

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