A STUDY ON THE CHANGES ON TEACHERS' KNOWLEDGE AND BELIEFS AFTER A WORKSHOP BASED ON MATHEMATICS EDUCATION SOFTWARE, BY RELYING ON FUZZY METHOD

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In this paper, the effect of holding a math training workshop using GeoGebra software has been studied on the changes on teachers' knowledge and beliefs . The selected sample is 40 male and female teachers in Iran. Before and after the intervention were administered a pre and post questionnaire with two components: TPACK knowledge and teachers' beliefs. Fuzzy logic and Fuzzy TOPSIS methods were used to analyze the data. The results of this method showed a significant difference between the results before and after the workshop.

Keywords: Fuzzy analysis; Geogebra; Geometry; Teachers' beliefs; Technological pedagogical content knowledge

Un estudio de los cambios en el conocimiento y creencias de docentes, después de un taller con software educativo matemático, analizado mediante el método Fuzzy

El estudio contempla el efecto de un taller de matemáticas con GeoGebra sobre los cambios en el conocimiento y las creencias de los docentes. La muestra seleccionada considera 40 docentes en Irán. Se administró un cuestionario, antes y después de la intervención, enfocado en dos componentes: el TPACK y las creencias. Los datos se analizaron aplicando herramientas del método Fuzzy mediante el cual se evidencia una diferencia significativa entre los resultados antes y después del taller.

Términos clave: Análisis Fuzzy; Conocimiento tecnológico pedagógico del contenido (TPACK); Creencias de los docentes; Geogebra; Geometría

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Recent analysis of the findings of the TIMSS[1] test indicate that performance of most Iranian students in elementary school mathematics and science courses is not totally appropriate. They did not have the ability to respond to the applied, judgmental, and combined problems, and they were not in ideal and international standard level on making hypothesis and problem-solving skills (Martin, Mullis, Foy, & Stanco, 2012). One of the reasons for the Iranian students' failure is related to the teaching methodology and the approach of their teachers. Since there is a weakness in the curriculum and the like, the teacher can overcome them with the theoretical and practical knowledge that he has learned. The basis of teacher knowledge is grounded on two types of knowledge: content knowledge (what should be taught) and teaching skills (how to teach). Pre-service teachers often learn how to teach different subjects at teacher training centers and universities, but it should be claimed that compared to the student and learner community, the number of teachers who enter the education system from teacher education centers is very low. Moreover, if the pre-service teachers in these centers become familiar with teaching methods as the traditional teacher-centered method, their learning will not last long. How the teachers who have not learnt the mathematics teaching method or the teachers who have been trained by a passive method of teaching (teacher-centered method) could be able to teach students mathematics concepts in an efficient way? The use of active methods (non-traditional methods) is necessary not only for teaching students, but also for training teachers. As students learn social interaction and personal thinking through active exploration, teachers also learn more by experimentation, constructivist learning, and constructivist teaching. This process provides them with new knowledge acquiring opportunities (Aghazadeh, 2009).

Technological Knowledge and Teacher's Beliefs

Currently, the use of technology in education is common and many educational environments are equipped with technology and global networks. However, to integrate technology into the curriculum, teachers need to be trained in a highly specialized way. The most of teachers do not have enough information about the correct use of new technologies (software-hardware) and often they use them only for presenting the information (Karami, Karami, & Attaran, 2012). In previous studies, different models have been proposed to integrate technology into teaching. Among them, technological pedagogical content knowledge (TPACK), introduced by Mishra and Koehler (2006), in which it points to the effective use of technology in education. Based on TPACK, technology is not an added factor to curriculum elements. This model is useful for creating goals, methods, materials and determining a kind of flexible assessment that can fit with different learners. In fact, this structure is a new model that depends on the

[1] TIMSS: Trends in International Mathematics and Science Study.

varied and flexible nature of the new media. This model can be adapted to any kind of curriculum design according to the way students learn. At the same time, it takes the individual differences among students into account (McAnear, 2009). According to Mishra and Koehler (2006), the structure of TPACK model is composed of the following components:

Pedagogical knowledge (PK). A deep knowledge of processes, approaches, and teaching and learning methods. This knowledge includes the ways of setting up the methods to achieve the educational goals. In fact, it involves a general understanding of the way students learn, classroom management, development, implementation, and evaluation of the curriculum.

Technological knowledge (TK). Technology in its current sense includes knowledge of the ways of installing, running and using a variety of computer-related software and hardware, which includes the skills, such as the skill in system administration and the use of tools such as Word and Internet.

Content knowledge (CK). This knowledge is the disciplinary content that teachers should teach and students should learn (Mishra & Koehler, 2006).

Technological pedagogical knowledge (TPK). It is a knowledge associated with the recognition of a variety of technologies that can be used in learning and teaching situations, as well as the knowledge of the way teaching method might change because of using the existing technologies.

Technological content knowledge (TCK). This knowledge demonstrates how the specific educational contents and technology mutually relate to each other. In fact, teachers need to know not only about the content they teach, but also need to know how the content changes according to the technological context, because today, technology tools can change the structures of the educational subjects.

Pedagogical content knowledge (PCK). This knowledge determines which educational approach is compatible with any content type.

Technological pedagogical content knowledge (TPCK). This knowledge is made of three types of content knowledge, pedagogical knowledge, and technological knowledge, and goes beyond all these knowledge types. This knowledge requires an in-depth understanding of the above concepts which effectively uses technology to construct teaching content. In other words, this knowledge makes it possible to solve educational problems using technology. After a while, the word TPCK changed to TPACK for the ease of pronunciation.

From the previous study on TPACK recognition and explanation, it can be concluded that this model with a theoretical support in explaining the integration of technology, is a promising path into the successful integration of technology into the curriculum. Nowadays, TPACK has been used as a theory, model, educational approach, and an approach to assess the knowledge of technology integration in educational institutions; and in many studies, it has shown that it abilities to enhance learning (Wang, 2015). The capabilities of this educational model are to the extent that Hoang (2015) considers this model essential to take advantage of the combined learning. Hence, equipping teachers with TPACK knowledge to enable them to integrate technology into teaching has been raised as one of the important programs in many educational systems of the world. Despite the little time passed after its introduction, many studies have been conducted on strategies, approaches and factors influencing its progress. In a school, although some teacher work in teaching process personally, this process requires collective and environmental factors. Therefore, the teacher's personal beliefs might be influenced by collective and environmental factors. In this case, one of the most important elements of the teaching-learning process is the teachers' beliefs, which play a decisive and undeniable role in achieving the goals and missions of education in quantitative and qualitative terms. One of the important criteria for effective teacher performance is their ability to control learners' behaviors and ultimately controlling the teaching and learning process (Hardy & Lanan, 2006).

In the process of education, teachers' beliefs play an important role in choosing their teaching methods. Hofer (2001) defined (teaching) belief as the choice of behavioral approach during teaching-learning process, which derives from values, beliefs, characteristics, aspirations, and history-individual and social culture. Teacher's behaviors in classroom are called "teaching beliefs" which is considered as one of the effective and important factors in teaching, because it determines the type of teacher training tasks and it is the basis for the learners' classroom activities (Shabani, 2011). Teacher beliefs in teaching are strategies that teachers use in their teaching process in order to affect the learners' mental-practical processes, and ultimately, learners can better adapt to their living environment or can act more effectively and flourish all their personality aspects (Hofer & Pintrich, 1997). Hofer and Pintrich (1997) defined belief as a coherent and fully consistent system of teacher education activities and learners' learning activities that take place for specific educational purposes. Recognizing teacher beliefs is necessary in achieving educational goals in any educational system. Because knowing the beliefs of teachers can determine the vital elements of the teaching-learning process for teacher and learner.

Since in this research two categories of high school teachers' knowledge and beliefs before and after the software training are considered, we attempt to investigate the changes in teachers' knowledge and beliefs after teaching, by scrutinizing and analyzing the teachers' knowledge and beliefs categories in using educational mathematics software. Smith, Kim, and McIntyre (2016) showed that technologies are useless in math classes and teachers need to overcome barriers to use it. Lack of adequate knowledge and lack of correct beliefs are the main obstacles in teachers' use of technology in teaching. They found that there is a relationship between teacher's beliefs about the nature of math, learning and teaching math, and the use of technology, content knowledge, pedagogical content knowledge, and technological pedagogical content knowledge. Güneş and Bahçivan (2016) focused on teachers' technological pedagogical content knowledge in a conceptual and perceptual system. The beliefs of pre-service teachers were also examined. The results showed that the system of epistemological beliefs of pre-service teachers was related to their TPACK. Therefore, the researcher sought to answer the question about the extent of using the educational mathematics software based on the growth of the high school teachers' knowledge; and the extent of using educational mathematics software, based on the change in the high school teachers' beliefs.

DYNAMIC SOFTWARE

According to Hohenwarter and Preiner (2007), Markus Hohenwarter developed GeoGebra which is a free, open-code dynamic software in mathematics field, and is used for both learning and teaching math in all levels of education. GeoGebra software has an entirely compacted, easy-to-use and connected environment to represent calculus, algebra, and geometry features. In order words, this software outspreads the notions of dynamic geometry to the mathematical analysis and algebra areas. GeoGebra, specially designed for educational purposes, can help students understand the research-based, problem-based, and experimental learning of mathematics, both at home and in the classroom. Students can enhance their cognitive abilities in the best way by using an interactive geometric system and a computer algebra system, simultaneously. GeoGebra as dynamic geometry software reinforces the structures with lines, points, and all conic pieces. Typical features are also provided by this software for a Computer Algebra System, like finding integrals and derivates of the entered functions, direct input of coordinates and equations, and finding main points of functions (extrema, roots, inflection and local points of functions), hence, making GeoGebra a good choice for various demonstrations of mathematical objects. The basic idea of GeoGebra's interactive program is to present two demonstrations of each mathematical object in its graphics and algebra windows. Any change of the objects in one of these windows, leads to an instant update in its demonstration in the other window. Dynamic geometry software (for example, Cabri Geometry, Geometer's Sketchpad) and Computer algebra systems (such as Maple, Mathematica) are among the dominant technological instruments for mathematics education. Many research findings recommend the use of these software packages to encourage visualization, experimentation, and discovery in traditional methods of mathematical teaching. In contrast, according to Ruthven and Hennessy (2004), some studies indicate that the main problem of many teachers is with the way of providing the required technology for the successful addition of technology into teaching. Hence, the software packet

GeoGebra is the recommended solution for using technology in the college level mathematics teaching. The benefits of applying GeoGebra software are as follow:

- ♦ GeoGebra is more user-friendly software, compared to a graph calculator. GeoGebra provides help, multilingual menus, commands, and easy-to-use program.
- ♦ GeoGebra encourages guided and experimental discovery learning, class` projects in mathematics, and multiple demonstrations.
- ♦ By adapting the program interactions (like line thickness, color, font size, language, quality of graphics, line style, coordinates, and other characteristics), the learners can personalize their own productions.
- ♦ GeoGebra software was developed to help learners gain a better grasp of the mathematical concepts. Using sliders and simply through dragging "free" objects around the drawing plane, learners can easily manipulate the variables.
- ♦ By applying a method of manipulating free mathematical objects, Students can make changes, and learn the way through which the dependent objects will be affected. Thus, through examining mathematical relations in a dynamic fashion, students will have the problem-solving opportunity.
- ♦ As Dubinsky and Schwingendorf (2004) stated, cooperative learning is an appropriate framework for a mathematics course. A task-based interactive classroom should take the place of lecturing. The main role of teaching is to create situations that will help learners in developing their process of building the required mental constructions, not to explain, give speech, or then try to "transfer" mathematical knowledge.

According to Prusak, Hershkowitz, and Schwarz (2012), the dynamic geometry instruments have been developed by mathematics teachers to encourage reasoning in geometry and to allow inquiry-based environment. Moreover, in many studies including the ones by Baccaglini-Frank and Mariotti (2010), and Arzarello, Olivero, Paolo, and Robutti (2002), researchers underlined the significance of dragging in conjecturing. For example, according to Arzarello, Olivero, Paolo, and Robutti (2002), dragging encourages exploring and conjecturing because individuals will have the opportunity of observing the fixed properties after changing the forms. As Arzarello, Olivero, Paolo, and Robutti (2002) claimed, gaining immediate feedbacks is helpful for proving and discovering constant properties of drawings. According to Healy and Hoyles (2001), dynamic geometry software enables students to create and experiment with geometrical objects to make clarifications and inferences. Understanding the benefits and disadvantages of using dynamic geometry software (GeoGebra) is necessary because the goal of using this software is to improve the quality of teaching. Arzarello, Olivero, Paolo, and Robutti (2002) claimed that both teacher and technology lead to an educational change.

METHOD

In this study, a semi-experimental method is used. The researcher tries to make his method in conducting the study closer to the experimental method by identifying the variables and expanding the necessary knowledge. However, he can use a method called "One group Pre-test, Post-test" to examine and study the situation. In this method, the dependent variable is measured before and after the implementation of the independent variable.

Participants

The statistical population of the study included both male and female teachers in Bojnourd, Northern Khorasan Province, Iran. Therefore, according to the available samples, we considered 40 of them who were teaching in the 7th and 8th grade. These teachers were teaching in in-service training courses and their age were in the range of 27 to 32, and considering the field of study, most of them were studied in the field of mathematics teaching and their work experience was ranged from 4 up to 7 years.

Instruments

According to the research question design and to collect the data, it is necessary to determine the research tools. In this study, a questionnaire was used. A questionnaire is a tool or instrument for collecting data from the selected sample members. Preparing a questionnaire is a scientific and precise implementation that is done by the researcher or the experts in this field. The used questionnaire consisted of three main parts: the personal characteristics of the teachers, the growth of teachers' technology knowledge, and teachers' beliefs. This questionnaire was developed by Zambak (2014), based on theoretical foundations, that each of its sections contains specific components and items: in the individual profile section (teachers' demographic section), age range, field of study before the university entrance, and work experiences has been raised. In the section of the teachers' growth in technological knowledge, 56 items were designed in five Likert scale. The description of the components and items are indicated in Table 1.

Table 1
Components and Items of Teachers' Knowledge and Beliefs

Components	Items
Teachers' knowledge items	
Technological knowledge (TK)	1-6
Content Knowledge (CK)	7-18
Pedagogical knowledge(PK)	19-25
Pedagogical content knowledge (PCK)	26-29
Technological content knowledge (TCK)	30-33
Technological pedagogical knowledge (TPK)	34-42
Technological pedagogical content knowledge (TPACK)	43-46
Teachers' beliefs items	
Nature of mathematics	47-48
The way of teaching mathematics	49-50
Viewpoints toward technology	51-52
Experience of teaching via dynamic mathematics software	53-54
The way of using in the direction of understanding or lack of understanding the mathematics	55-56

In the teacher beliefs section, there are 14 descriptive items, which include the item numbers from 57 to 70. This questionnaire distributed among the teachers before and after the educational intervention (Dynamic GeoGebra software workshop). It should be noted that these items corresponding to the fuzziness of the questionnaires turned to 70 items. In the validity and reliability section with CVI index, for most of the items the amount of 0.80 was obtained, which according to CVI index the content validity of the questionnaire was approved. The Cronbach's alpha was used to analyze and measure the reliability of the questionnaire. The obtained reliability was 0.72. Since these values are higher than 0.70, therefore the questionnaire is reliable.

Implementing teaching with Geogebra

GeoGebra is one of the dynamic geometry software collections combining geometry, algebra, and calculus concepts. GeoGebra allows you to display objects in three ways: graphics (such as points and function diagrams), algebraic (such as coordinates of points and equations) and in spreadsheet cells. All methods of displaying an object are related to one another, and an object can be automatically transformed into other demonstrative methods, regardless of how it is created. After identifying the target group which included the teachers working in the current academic year at the seventh and eighth grades (see Table 2) the questionnaire of the teacher's knowledge and beliefs distributed among the teachers. Then, planning for implementing the teachings based on the dynamic software including GeoGebra took place. At the beginning of the sessions, some self-studying tutorials were given to the experimental group and, according to these topics, the educational workshops were conducted.

Table 2
Educational Topics of the Seventh and Eighth Grade Books in the Educational Workshop

Seventh-grade Book	Eighth-grade Book
Line segment, angle, drawing triangle and geometric shapes (geometry and reasoning).	Symmetry, parallel, foursquare, internal and external angles (polygons).
Directed line segment, equal vector, coordinate (vector and coordinates).	Line and circle, central and inscribed angles (circle).
Triangle and its components, parallel and diagonal lines (geometric and parallel drawings).	Pythagoras, modular arithmetic (triangle)

The training course held in 12 sessions, twice a week; and teaching and practice sessions lasted for three months. Finally, after holding the training course (workshop), the questionnaires distributed among the teachers again.

DATA ANALYSIS

In this section, a questionnaire was first designed and distributed in an educational workshop among 40 participants. Then, the way of using GeoGebra software was taught. At the end of the course, once again, the questionnaires were distributed among the teachers. Then, using Fuzzy logic and Fuzzy TOPSIS method, data were analyzed. Fuzzy TOPSIS method is one of the techniques used in Fuzzy Multi Criteria Decision Making (FMCDM). In this decision-making method, there are several options and a number of criteria for decision making that according to the criteria, the options must be ranked, or a performance score should be allocated to each of them. The general philosophy of the Fuzzy TOPSIS method is that using the available options, two hypothetical options will be defined. One of these options is a set of the best values observed in the decision matrix. This option is called the positive ideal (best possible state). In addition, another hypothetical option is defined which includes the worst possible states. This is called the ideal negative option. The criteria can be of either positive or negative nature, and their measurement unit can also be different. The criteria for calculating scores in the TOPSIS method are that options be close to the positive ideal option and far from the negative ideal option, as much as possible. Accordingly, a score is calculated for each option and the options are ranked according to these scores. Exact and definite values are used for determining the weight of the criteria and ranking the options. In many cases, the human thinking is associated with uncertainty and this uncertainty is influential in decision making. As stated, in these cases, it is better to use fuzzy decision-making methods that the similar to fuzzy ideal option method is one of these methods. In this case, the elements of the decision matrix or the weight of the criteria, or both, are evaluated by the language variables which are provided by the fuzzy numbers. We have applied statements and computations along formulas for Fuzzy TOPSIS method (as see in following statements) from Zimmermann (1985) and Dubois and Prade (1980).

Fuzzy TOPSIS Method Algorithm

Step1. Developing a decision matrix

Given the number of criteria, the number of options, and the evaluation of all options for different criteria, the decision matrix is made up as (1):

$$\tilde{D} = \begin{pmatrix} \tilde{X}_{11} & \tilde{X}_{12} & \cdots & \tilde{X}_{1n} \\ \tilde{X}_{21} & \tilde{X}_{22} & \cdots & \tilde{X}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{X}_{m1} & \tilde{X}_{m2} & \cdots & \tilde{X}_{mn} \end{pmatrix} \qquad (1)$$

If the triangular fuzzy numbers are used, $\tilde{X}_y = (a_y, b_y, c_y)$ is the function of the option i (i=1,2,…,m) related to the j (j=1,2,…,n) criterion. If the decision-making committee has $k-$members and the fuzzy ranking of the k^th^ decision maker is $\tilde{X}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$ (triangular fuzzy number) for i=1,2,…,m and j=1,2,…,n, according to the criteria, the options' fuzzy combinational ranking of $\tilde{X}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ can be obtained based on the equations (2):

$$a_{ij} = \min_k \left\{a_{ijk}\right\}$$
$$b_{ij} = \frac{\sum_{k=1}^{K} b_{ijk}}{k} \qquad (2)$$
$$c_{ij} = \max_k \left\{c_{ijk}\right\}$$

In the fuzzy TOPSIS method, if the total number of decision makers are $k-$ members, and the fuzzy prioritization of the $k-$ th decision maker for

$i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$, is $x_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$ (triangular fuzzy number), the combined fuzzy prioritization of the options $x_{ij} = (a_{ij}, b_{ij}, c_{ij})$ can be justified according to the formulas (2). Pay attention to the following points and examples, to further clarify the issue. In the Fuzzy TOPSIS method, there are a number of options and a number of criteria for decision making, that the options should be prioritize according to criteria, and this criterion can have positive or negative aspects, which in this paper, all criteria are considered positive. It can also have weight.

Step 2. Determining the criteria weights matrix

In this step, the importance coefficient of the different criteria in decision making is defined as (3):

$$\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n) \quad (3)$$

In a way that if the triangular fuzzy numbers are used, each of the components of w_j (weight of each criterion) will be defined in the form of $\tilde{w}_j = (\tilde{w}_{j1}, \tilde{w}_{j2}, \tilde{w}_{j3})$ and in case the trapezoidal fuzzy numbers are used, each of the components of w_j will be defined in the form of $\tilde{w}_j = (\tilde{w}_{j1}, \tilde{w}_{j2}, \tilde{w}_{j3}, \tilde{w}_{j4})$. If the decision-making committee has $k-$members and the importance coefficient of the $k-$th decision maker is $\tilde{w}_{jk} = (\tilde{w}_{jk1}, \tilde{w}_{jk2}, \tilde{w}_{jk3})$ (triangular fuzzy number) for $j = 1,2,\ldots,n$, the fuzzy combinational ranking $\tilde{w}_j = (\tilde{w}_{j1}, \tilde{w}_{j2}, \tilde{w}_{j3})$ can be obtained from the equations (4):

$$w_{j1} = \min_k \{w_{jk1}\}$$
$$w_{j3} = \min_k \{w_{jk3}\} \qquad (4)$$
$$w_{j2} = \frac{\sum_{k=1}^{K} w_{jk2}}{k}$$

Step 3: Non-scaling the fuzzy decision matrix

When the x_{ij}s are in fuzzy form, the r_{ij}s will also be fuzzy. At this stage of the linear scale changes for converting the scale of different criteria into comparable scale. If the fuzzy numbers are in triangular form, the non-scaled decision matrix entries for the positive and negative criteria are respectively calculated from the equations (5):

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+} \right) (+)$$
$$\tilde{r}_{ij} = \left(\frac{a_j^-}{a_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{c_{ij}} \right) (-) \qquad (5)$$

That in these equations (6):

$$\left\{ \begin{array}{l} c_j^+ = \max\limits_i c_{ij} \\ a_j^- = \min\limits_i a_{ij} \end{array} \right\} \qquad (6)$$

Therefore, the non-scaled Fuzzy decision matrix ($\tilde{R}$) is obtained as follows $\tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n}$ $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$, or:

$$\tilde{R} = \begin{pmatrix} \tilde{r}_{11} & \cdots & \tilde{r}_{1j} & \cdots & \tilde{r}_{1n} \\ \vdots & & \vdots & & \vdots \\ \tilde{r}_{i1} & \cdots & \tilde{r}_{ij} & \cdots & \tilde{r}_{in} \\ \vdots & & \vdots & & \vdots \\ \tilde{r}_{m1} & \cdots & \tilde{r}_{mj} & \cdots & \tilde{r}_{mn} \end{pmatrix} \qquad (7)$$

Where m denotes the number of options and n denotes the number of criteria.

Positive or negative criteria may be considered here. For example, punishment of the students or the teachers' moral problems can be attributed to the negative criterion. However, here, the questions are designed in such a way that all criteria are positive. Therefore, the response matrix will remain without change. Following the response to this section, it is worth noting that the positive and negative criteria are in such a way that appear in the form of qualitative numbers and based on the definition of triangular fuzzy numbers (according to the expert's opinion), they can be considered as the low numbers in a scale of 10 points, that is up to 4.

But this does not mean that any number which is part of the low numbers between 0 and 10 is a negative criterion, and it goes back to the essence of the question. If in the concept of the question, there are negative aspects affecting the goal, it is called negative criterion, and if there are positive aspects affecting the goal, it is called positive criterion.

Step 4: Determining the weighted fuzzy decision matrix

Considering the weight of different criteria, the weighted fuzzy decision matrix is obtained by multiplying the importance coefficient for each criterion in the fuzzy unscaled matrix as (8):

$$\tilde{v}_{ij} = (\tilde{r}_{ij} \cdot \tilde{w}_j) \qquad (8)$$

Where $\tilde{w}_j$ indicates the importance coefficient of the criterion c_j.

Therefore, the weighted fuzzy decision matrix will be as follows:
$\tilde{V} = \left(\tilde{v}_{ij}\right)_{m\times n}$; $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$ or:

$$\tilde{V} = \begin{pmatrix} \tilde{v}_{11} & \cdots & \tilde{v}_{1j} & \cdots & \tilde{v}_{1n} \\ \vdots & & \vdots & & \vdots \\ \tilde{v}_{i1} & \cdots & \tilde{v}_{ij} & \cdots & \tilde{v}_{in} \\ \vdots & & \vdots & & \vdots \\ \tilde{v}_{m1} & \cdots & \tilde{v}_{mj} & \cdots & \tilde{v}_{mn} \end{pmatrix} \qquad (9)$$

If the fuzzy numbers be in triangular form, we will have the following equations (10) for the criteria with positive and negative aspects:

$$\begin{aligned} \tilde{v}_{ij} &= \tilde{r}_{ij} \cdot \tilde{w}_j = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right) \cdot \left(w_{j1}, w_2, w_{j3}\right) = \left(\frac{a_{ij}}{c_j^+} w_{j1}, \frac{b_{ij}}{c_j^+} w_{j2}, \frac{c_{ij}}{c_j^+} w_{j3}\right) \\ \tilde{v}_{ij} &= \tilde{r}_{ij} \cdot \tilde{w}_j = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right) \cdot \left(w_{j1}, w_2, w_{j3}\right) = \left(\frac{a_j^-}{c_{ij}} w_{j1}, \frac{a_j^-}{b_{ij}} w_{j2}, \frac{a_j^-}{a_{ij}} w_{j3}\right) \end{aligned} \qquad (10)$$

When the options have criteria and these criteria have weight and a weight are defined for them, positive and negative criteria appear for a series of used data. Since fuzzy numbers have their own particular conditions, and can be applied according to those conditions, weights do not distribute equally on these numbers. Consequently, if positive and negative criteria show up, then they will surely satisfy the conditions of the fuzzy numbers and will be applied to them. Here, the triangular fuzzy numbers $x = (x_1, x_2, x_3)$ were used, under the condition of $x_1 < x_2 < x_3$, and this always holds true.

Step 5: Finding the Fuzzy Positive Ideal Solution (FPIS, A+) and the Fuzzy Negative Ideal Solution (FNIS, A-)

The fuzzy positive ideal solution and the fuzzy negative ideal solution are defined as (11):

$$\begin{aligned} A^+ &= \left\{\tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_n^+\right\} \\ A^- &= \left\{\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-\right\} \end{aligned} \qquad (11)$$

Where $\tilde{v}_1^+$ is the best amount of criterion i from all the options and $\tilde{v}_1^-$ is the worst amount of criterion i from all the options. These values are obtained from the following equations (12):

$$\begin{aligned} \tilde{v}_j^+ &= \max_i \left\{ \tilde{v}_{ij3} \right\} \\ \tilde{v}_j^- &= \min_i \left\{ \tilde{v}_{ij1} \right\} \end{aligned} \quad i = 1,2,\ldots m; j = 1,2,\ldots n \qquad (12)$$

The options that appear in A^+ and A^-, respectively represent the options of “entirely better” and “entirely worse.”

Step 6: Calculating the distance from the ideal and anti-ideal fuzzy solution

$$\begin{aligned} S_i^+ &= \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+), \\ S_i^- &= \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \end{aligned} \quad j = 1,2,\ldots,m \quad (13)$$

$d_v(\cdot,\cdot)$ is the distance between two fuzzy numbers that if $N_1 = (a_1, b_1, c_1)$ and $N_2 = (a_2, b_2, c_2)$ be two triangular fuzzy numbers, the distance between the two numbers will equal to:

$$d_v(N_1, N_2) = \sqrt{\frac{1}{3}\left[(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2\right]} \qquad (14)$$

It should be noted that $d_v(\tilde{v}_{ij}, \tilde{v}_j^+)$ and $d_v(\tilde{v}_{ij}, \tilde{v}_j^-)$ are crispy numbers.

Step 7: Calculating the similarity index
The similarity index is calculated from the following equation (15):

$$CC_i = \frac{s_i^-}{s_i^+ + s_i^-}, i = 1,2,\ldots,m \qquad (15)$$

Step 8. Ranking the options
At this stage, according to the amount of similarity index, the options are ranked so that the options with more similarity index are prioritized.

In answering to the relevant letters, the answers to the questions are in qualitative option types that are presented in the form of words such as very high, high, average, always, often, and so on. Given the value of each of these words, appropriate Fuzzy numbers corresponding to them are considered, which indicates their qualities. For ease of operation and without any intersection in the thread, all fuzzy numbers are represented in triangular forms, that the support of each is defined in the interval of [0,1]. Obviously, this is done with the normalization process. The most important point is the creation of a membership function and a membership degree for the members of the number supports, which show their quality. The fuzzy numbers associated with the above language variables are listed below. For the described questions designed in the questionnaire, the responses were first reviewed and analyzed before and after the workshop, and finally, it was seen that the answers were in qualitative expressions, in the form of linguistic and colloquial terms. According to the value of each of these Linguistic words which were corresponding to the responses of teachers, the appropriate fuzzy numbers which indicated their quality, were designed corresponding to them. For the convenience of work and without damaging the whole subject, all fuzzy numbers are presented in triangular form. Now we obtain the indices based on positive and negative ideal square root (see Table 3).

For analyzing the results, we have two methods. One is a statistical method, and the other one is a mathematical method, that we have used the fuzzy TOPSIS method in this study which is a very powerful method for ranking. Since human thoughts always influence decision making with uncertainty, the use of a fuzzy method such as fuzzy TOPSIS method is much stronger than statistical methods, because the nature of uncertainty can be applied to the designing of the question, and teachers can also apply their opinions in responding to the questions in a better way. And by applying this method in Table 3, all 70 criteria studied in our article have all grown together, which is a very strong reason, considering the existence of positive and negative ideals.

According to the extracted indices of the Fuzzy TOPSIS method, that their differences have been positively mentioned in the last column of the above table, the results before the workshop and after the workshop have significant differences, and this difference indicates that after holding the workshop on technological knowledge, beliefs and teaching practices, the teachers' ideology has grown dramatically.

Table 3
Indices Based on Positive and Negative Square Root Before and After the Educational Workshop

Items	Index before workshop	Index after workshop	Difference
1	0.509715	0.532668	0.022953
2	0.492151	0.548333	0.056182
3	0.541302	0.545784	0.004482
4	0.434213	0.534085	0.099872
5	0.474267	0.517792	0.043525
6	0.420662	0.566037	0.145375
7	0.43391	0.512014	0.078104
8	0.490372	0.551817	0.061445

Table 3
Indices Based on Positive and Negative Square Root Before and After the Educational Workshop

Items	Index before workshop	Index after workshop	Difference
9	0.465968	0.51281	0.046842
10	0.511191	0.537606	0.026415
11	0.403513	0.51374	0.110227
12	0.476739	0.527699	0.05096
13	0.465512	0.516624	0.051112
14	0.508708	0.513561	0.004853
15	0.40818	0.521994	0.113814
16	0.412183	0.546723	0.13454
17	0.506383	0.545795	0.039412
18	0.41488	0.507503	0.092623
19	0.467269	0.484936	0.017667
20	0.447932	0.503584	0.055652
21	0.521196	0.545624	0.024428
22	0.421746	0.53616	0.114414
23	0.421887	0.57715	0.155263
24	0.48324	0.611158	0.127918
25	0.465874	0.60575	0.139876
26	0.51595	0.518927	0.002977
27	0.459106	0.469585	0.010479
28	0.459538	0.484218	0.02468
29	0.454082	0.536075	0.081993
30	0.471479	0.548401	0.076922
31	0.415723	0.544048	0.128325
32	0.381345	0.492387	0.111042
33	0.510802	0.521193	0.010391
34	0.419713	0.519938	0.100225
35	0.448647	0.536947	0.0883
36	0.399473	0.535218	0.135745
37	0.448132	0.534338	0.086206
38	0.445045	0.508826	0.063781
39	0.517768	0.521196	0.003428
40	0.422397	0.533513	0.111116
41	0.355955	0.535361	0.179406
42	0.415367	0.491033	0.075666
43	0.454268	0.528943	0.074675
44	0.476792	0.556029	0.079237
45	0.52059	0.564451	0.043861
46	0.445841	0.561305	0.115464
47	0.463873	0.537909	0.074036
48	0.425736	0.543989	0.118253

Table 3

Indices Based on Positive and Negative Square Root Before and After the Educational Workshop

Items	Index before workshop	Index after workshop	Difference
49	0.523463	0.530368	0.006905
50	0.420162	0.516911	0.096749
51	0.466935	0.477939	0.011004
52	0.473626	0.537635	0.064009
53	0.478746	0.491645	0.012899
54	0.478861	0.571631	0.09277
55	0.430038	0.534676	0.104638
56	0.503257	0.529954	0.026697
57	0.421666	0.532856	0.11119
58	0.478254	0.484187	0.005933
59	0.523005	0.534996	0.011991
60	0.502328	0.520557	0.018229
61	0.478306	0.568485	0.090179
62	0.449677	0.532096	0.082419
63	0.516352	0.533586	0.017234
64	0.471388	0.489281	0.017893
65	0.497201	0.58739	0.090189
66	0.513095	0.535349	0.022254
67	0.471699	0.588137	0.116438
68	0.513996	0.526374	0.012378
69	0.473626	0.499806	0.02618
70	0.454946	0.569959	0.115013

CONCLUSION

Teaching is one of the most challenging and important jobs. Teachers play a vital role in facilitating the efficient and effective learning in the teaching-learning process, at present and in the future. They help the learners easily move from the family environment to the unfamiliar school environment, bring the outside world into the classroom, and transfer the classroom to the outside world. Technologies have reduced educational problems to some extent in the current era; however, it has certainly not provided the welfare of teachers. Since the goals of education have become more complicated and individual differences have accordingly increased, training some skills is no more enough. Teachers are expected to help students reach the highest levels of cognitive domains, including creativity, learning, and relationship between the findings and, most importantly, the way of learning new knowledge and applying it to new situations. Learners' perceptions of the nature of learning have changed. In order for learning to take place, learners should be active, learning should be meaningful and real, and learning environment should be challenging. Knowledge is expanding in the 21st century, and a large part of it is available to teachers and students. This fact gives teachers an important responsibility, updates their knowledge, and exposes them to the global and modern networks.

The limited research on the integration of learning and technology, the weaknesses in content knowledge, teachers' teaching skills in high school math concepts, the lack of a model for incorporating technology into the math curriculum, the benefits of student-centered learning, the benefits of using computers in education, the interest and belief of teachers in using computers in the curriculum, and the teaching of computer use in the curriculum for teachers in the field of mathematics are among the various reasons that made it necessary and essential to conduct this research. This research aimed at developing the teachers' knowledge and beliefs in teaching high school mathematical concepts through integration of learning with math software. Mathematics teachers have different beliefs about the nature of mathematics as a scientific discipline and a school subject. Knowing that at least some teachers have different beliefs about school mathematics and mathematics as a scientific discipline can explain the apparent inconsistencies between teacher's beliefs about mathematics and its teaching-learning process and, consequently, their impact on teaching.

One of the benefits of using the GeoGebra teaching was that the teachers in the GeoGebra group could make dynamic and very precise drawings for the given tasks. Thus, they could go on forming implications and explanations, become more confident in their reasoning, and easily see the relationships. Yet, in the Paper-pencil group, due to their incorrect drawings, it could be inferred that whether they made wrong conjectures, or they were not sure about their conjectures. Another benefit of using the GeoGebra in reasoning can be the dragging option that makes exploring the relationships and preserving some geometrical properties of a figure possible. The GeoGebra group supported conceptual understanding by making the generalizations easy, and by using the dragging option successfully. Besides, they were more interested in discussing and finding different solutions to the problem. For example, if they noticed any relationship in an equilateral triangle, through dragging the vertices of the triangle and by changing its properties, they could check the validity of the relationship for other triangles. But in the Paper-pencil group, such opportunity was not provided for the subjects. They were busy drawing the new shape with several attempts on the paper, using the materials like compass, ruler, and protractor. Thus, the repetitive drawings made the concentration on the relationships between the fixed properties of different shapes difficult for the subjects leading to the decrease in their motivation for discussion. Saving the time is another benefit of the GeoGebra. Using different options of the GeoGebra, like drawing parallel line or perpendicular, measuring angle, drawing circle, measuring side, and so on, the subjects drew shapes quickly and easily. In the GeoGebra group, most of the subjects' time was spent over argumentation and reasoning rather than being spent on their drawings. Yet, drawing angle bisector, measuring angle, drawing circle with compass and so on were time-consuming for the Paper-pencil teaching. Sometimes, due to the use of wrong conjectures, they had to draw the shape again and many times stop the argumentation. In the same situation, the GeoGebra group members could go back on the screen using "Ctrl + Z" buttons and keep drawing on the same shape. Therefore, retaining the argumentation and saving the time made GeoGebra a useful program.

In the technology-based teaching or training, such as software training for teacher training courses, the strengths and weaknesses of the teachers' skills and abilities get more accurately identified. Some mathematics teachers might have relatively good abilities and skills in teaching based on software such as the GeoGebra, but they do not know when to use these abilities in the class while teaching the students. On the other hand, the discussion of the beliefs and knowledge about the non-abstract nature of the software for some teachers can flourish their latent abilities, which will be possible and take place when the training courses are provided for them. These training courses should be held as workshops by experts.

In most mathematical education research, statistical or descriptive methods are used to assess the performance and abilities of the learners under the teaching of a new intervention. In this study, the attempt was to show that qualitative results through can even be achieved using logical mathematical methods with mathematical reasoning. Therefore, the slight differences in the performance of mathematical teachers were assessed before and after the new intervention with a new approach such as fuzzy logic. In statistical methods such as SPSS, even the slight differences for each participant sample might not appear in both positive and negative terms. However, in methods such as fuzzy logic, such differences can be observed.

Human thoughts always affect decision-makings with uncertainty. That is why much attention has been paid to designing the teachers' answer sheets, and the questionnaire has been designed accordingly to cover this issue. Therefore, a powerful fuzzy tool has been used in this case. One of the strongest methods for decision-makings with uncertainty is the fuzzy TOPSIS method. One of the important advantages of this method is that both objective and subjective indicators and criteria can be used simultaneously. In this model, for mathematical calculations, all given ratios to the criteria must be of quantitative type and, if the given ratios to the criteria are of the qualitative type, the qualitative values should be converted into quantitative values.

Teachers should revise their beliefs to align with effective teaching. It will have an impact on classroom practices if teachers do not have such beliefs considering the way of their teaching and the way of learning by the students. Teachers are the representatives of curriculum and content. Hence, teachers' beliefs are the most required domain for further studies. To elaborate on this study, those who serve as education administrators, involve technology in their teaching, educate teachers, teach geometry, and develop curriculum should consider how to develop teachers' beliefs. Eventually, working with teachers to foster their beliefs about TPACK and mathematics may increase the use of teaching that leads to students' literacy progress in geometry. In this research, teachings were based on the GeoGebra software. The GeoGebra is a dynamic mathematical model that connects geometry, algebra, and arithmetic. In other words, this software is an interactive geometric system. You can create different shapes such as functions that can dynamically change, using dots, vectors, line segments, lines, polygons, and conic sections. Alternatively, the equations and coordinates of the points can be directly entered the software. Therefore, the GeoGebra can work with numeric, vector and points variables. This software also calculates the derivative and integral of functions and includes the commands such as root and extremum. The overall results, using the fuzzy method, showed that teachers saw changes in their knowledge and beliefs. Teachers' teaching methods are strongly dominated by their knowledge and beliefs, especially the knowledge of technology and the beliefs they have about teaching and learning. Therefore, it is evident that the teachers' technological knowledge and beliefs have an undeniable effect on the students' learning outcomes.

Recommendations

Hence, it seems that investing more in the field of teachers' professional development can be very fruitful. Because this will make the teachers reflect on their attitudes, beliefs, and philosophy towards teaching-learning, thus by changing the teachers' beliefs, their teaching method will also be affected by their teaching. Also, to introduce new theories and methods of teaching, including the use of technologies in the school and university curriculum, a suitable platform must be provided for the revival of scientific morale, preventing the translation and import system, to witness the prosperity of the production of science in the country. Based on the discussions and the results, four implicational suggestions for further research are listed below:

- Using different ways, the future studies should try to provide new definitions for the relationship between TPACK and beliefs.
- Through including different data types in similar cases, this study can be confirmed and repeated in a form of a more comprehensive data triangulation.
- Regarding teacher education, results reveal that the change in teacher candidates' teaching - learning conceptions and the increase in TPACK levels seem possible, if self-construal education be provided for these teachers.

- ♦ It is recommended that math teacher education programs should involve self-construal education. In this regard, program developers can use the help of the social psychologists.

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