MULTIMODALITY AND THE SEMIOTIC BUNDLE LENS: A CONSTRUCTIVE RESONANCE WITH THE THEORY OF OBJECTIFICATION

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The paper situates mathematics teaching-learning processes within a multimodal perspective and discusses a semiotic approach apt to seize this dimension, namely the Semiotic Bundle lens. This analytical tool considers the great variety of semiotic resources through which mathematical meanings emerge and evolve in the classroom, ranging from embodied ones such as gestures, to symbolic systems. In particular, the analysis considers them in a systemic and dynamic way. The theoretical account is illustrated by means of an example on children spatial conceptualization, carried out in kindergarten. The data analysis will constitute a background against which the connections with the Theory of Objectification will be highlighted, showing a constructive resonance between the two theories.

Keywords: Gestures; Multimodality; Semiotic Bundle; Semiotic game

La multimodalidad y el lente Semiotic Bundle: una resonancia constructiva con la teoría de la objetividad

El artículo sitúa los procesos de enseñanza-aprendizaje de las matemáticas dentro de una perspectiva multimodal y discute un enfoque semiótico apto para aprovechar esta dimensión llamada el lente Semiotic Bundle*. Esta herramienta analítica tiene en cuenta la gran variedad de recursos semióticos a través de los cuales los significados matemáticos emergen y evolucionan en el aula, como los gestos hasta los sistemas simbólicos. En particular, el análisis los considera de forma sistémica y dinámica. La explicación teórica se ilustra mediante un ejemplo de conceptualización espacial de niños, llevado a cabo en el jardín de infantes. El análisis de datos constituirá un trasfondo contra el cual se resaltarán las conexiones con la teoría de la objetivación, mostrando una resonancia constructiva entre las dos teorías.*

Términos clave: Gestos; Juego semiótico; Multimodalidad; Semiotic Bundle

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A funding father of mathematics education research already in the nineties drew attention to *processes* as a crucial issue in mathematics education research: “the use of and the emphasis on *processes* is a *didactical principle*” (Freudenthal, 1991, p. 87, emphasis as in the original). Considering in particular how teaching-learning processes develop in the classroom, a variety of actions and productions on the part of students and teacher can be observed, and also recorded with the technological devices available nowadays. Such actions and productions make use of resources of different nature: words (spoken or written), written representations, extra-linguistic ways of expression (gestures, gazes …), tools, and so on. Studies in different fields appear to converge in highlighting that such resources play an important role not only in communication, but also in the cognitive processes underpinning the production of mathematical meaning.

This assumption emerged from results in psychological, psycholinguistics, and neuroscientific studies, as well as in communication science and led to the emergence of the so-called *multimodal perspective* on mathematics teaching and learning (Arzarello, 2006). In this paper I will present the multimodal perspective and its roots and discuss a semiotic approach apt to seize this dimension, namely the Semiotic Bundle lens. The theoretical account will be illustrated by means of an example on children spatial conceptualization, carried out in kindergarten. The data analysis will constitute a background against which the connections with the Theory of Objectification will be highlighted, showing a constructive resonance between the two theories.

Multimodality in Teaching-Learning Mathematics

An important contribution to the development of multimodal approaches to mathematical thinking has been provided by the embodied cognition studies. At the turn of the millennium, the provocative essay Where Mathematis Comes From by George Lakoff and Rafael Núñez (Lakoff & Núñez, 2000) pointed out the crucial role of perceptual and bodily aspects on the formation of abstract concepts, including mathematical ones. The publication of the volume opened up a great debate in mathematics education in those years (see e.g. Research Forums at PME conferences) and prompted a reflection about the cognitive roots of mathematical ideas (e.g. see Tall, 2000).

Criticizing the platonic idealism and the Cartesian mind-body dualism, Lakoff and Núñez (2000) advocated that all kinds of ideas—included the most sophisticated mathematical ones—are founded on our bodily experiences and develop through metaphorical mechanisms. Their account brought about two claims: (a) Our concepts are structured by our body and our everyday functioning in the world; and (b) Human beings come to understand and know abstract concepts (included mathematics) through their sensory-motor experiences, which are largely *metaphorical* in nature. This is not a completely new idea, if we think to the psychological studies by Piaget, Vygotsky, and Montessori, and the philosophical ones by Husserl, Merleau-Ponty, within the phenomenological strand. What was new in embodiment, and constituted a constructive innovation beyond the radical criticism that characterized the publications, was the theorization about the cognitive mechanism through which the mathematical ideas were linked to everyday concepts, that is the *cognitive metaphor* (e.g., time conceptualized as space, sets conceptualized as containers).

Such an account appears to receive confirmation by recent neuroscientific results related to the discovery of the so-called "mirror neurons", which fire both when a subject does a certain action (e.g., grasping an object), as well as when he sees somebody else doing the same action. Other neurons are found to be "multimodal", firing when a subject does an action, or sees it, or hear its noise. Grounding on these results, Gallese and Lakoff (2005) provided a new theoretical account on how the brain works.

> *The modalities of action and perception are integrated at the level of the sensory-motor system and not via higher association areas. […] Multimodal integration has been found in many different locations in the brain, and we believe that it is the norm. That is, sensory modalities like vision, touch, hearing, and so on are actually integrated with each other and with motor control and planning.* (Gallese & Lakoff, 2005, p. 259, emphasis as in the original)

The term "multimodality" emerged thus in the neuroscientific field to indicate a feature of human cognition, opposed to "modularity" and stressing a deep integration between aspects that traditionally were considered as neuronal separated, such as action and perception. Furthermore, such integration would result crucial not only for motor control, but also for planning actions: An activity typical of what is generally understood as thinking.

Also studies on the role of gestures in communication suggest a sort of integration between different modalities: "One uses parallel sensory pathways, audio-oral and visuo-gestural, which interact in multimodal communication, that is, the ensemble of spoken linguistic, prosodic, intonational, gestural, postural, and facial activity that participants engage in when they 'talk'" (Calbris, 2011, p. 2).

But more surprising, many researchers in gesture studies point out the role of gestures also in cognition, in synergy with spoken language. According to Hostetter & Alibali (2008), gestures "emerge from perceptual and motor simulations that underlie embodied language and mental imagery" (p. 495) and are to be interpreted as "a natural expression of the simulated actions that underlie speaking and thinking" (p. 504).

If already Vygotsky had stressed the constitutive role of language in thinking, by saying that "thought is not merely expressed in words; it comes into existence with them" (Vygotsky, 1934/86, p. 218), psychological studies on gestures push in the direction of extending this constitutive role of language to the speech-gesture unity: "Gestures do not just reflect thought but have an impact on thought. Gestures, together with language, help constitute thought" (McNeill, 1992, p. 242, emphasis as in the original).

Finally, in the communication field, the term "multimodal" is used with reference to multiples modalities that we can exploit in order to communicate and express meanings to our interlocutors: Words, sounds, images, and so on (Kress, 2004). These dimensions are acquiring today increasing attention due to the diffusion of new technological affordances.

Drawing on the results coming from the research fields of embodiment, neuroscience, gestures, new media, the term "multimodality" has emerged in research in mathematics education to refer to the relevance and mutual co-existence of a variety of cognitive, material and perceptive modalities or resources in the mathematics teaching-learning processes, and more in general in mathematical thinking: "These resources or modalities include both oral and written symbolic communication as well as drawing, gesture, the manipulation of physical and electronic artifacts, and various kinds of bodily motion" (Radford, Edwards, & Arzarello, 2009, pp. 91-92).

Semiotic Approaches to Multimodality

In the last decades, attention to multimodal aspects of mathematics learning has been payed from different perspectives (Edwards, 2009; Nemirovksy, 2003; Radford, 2003; Roth, 2001). Some studies have adopted a semiotic lens and have considered gestures and embodied resources as part of the range of semiotic resources that students and teachers can exploit in their mathematical activities: they mostly may be referred to Radford's Objectification Theory and to Arzarello's Semiotic Bundle lens.

Taking a semiotic-cultural approach, Radford (2003) discussed the role of signs in learning mathematics as part of the more general process of what he called *knowledge objectification*, which involve "those actions aimed at bringing or throwing something in front of somebody or at making something visible to the view" (p. 40). Objectifying is therefore related to noticing and to making something apparent—e.g. a certain aspect of a concrete object, like its color, its size, or a property of a mathematical object. In particular, in a social, interpersonal perspective, objectifying knowledge means making "things and relations apparent in their universe of discourse" (Radford, Demers, Guzmán, & Cerulli, 2003, p. 56).

In order to account for the complexity of processes of objectification in mathematics teaching and learning situations, Radford pointed out that focusing only on the semiotic register of representations (in the sense of Duval, 2006) is not enough. Following Vygotsky (Vygotsky, 1931/78), he took a wider perspective on signs and considered them in their functional role as psychological tools that allow the subject for reflections and planning actions, and as cultural mediators. Specifically, without denying or minimizing the pedagogical and epistemological role of representations and the huge importance ascribed to writing in our culture, Radford claimed that:

> *Processes of knowledge production are embedded in systems of activity that include other physical and sensual means of objectification than writing (like tools and speech) and that give a corporeal and tangible form to knowledge as well. Within this perspective and from a psychological viewpoint, the objectification of mathematical objects appears linked to the individuals' mediated and reflexive efforts aimed at the attainment of the goal of their activity. To arrive at it, usually the individuals have recourse to a broad set of means. They may manipulate objects (such as plastic blocks or chronometers), make drawings, employ gestures, write marks, use linguistic classificatory categories, or make use of analogies, metaphors, metonymies, and so on* (Radford, 2003, p. 41).

In knowledge objectification, to make something apparent learners and teachers make recourse to signs and artifacts of different sorts (mathematical symbols, graphs, words, gestures, calculators, and so on), which have been called *semiotic means of objectification*. Following Radford (2003):

> *These objects, tools, linguistic devices, and signs that individuals intentionally use in social meaning-making processes to achieve a stable form of awareness, to make apparent their intentions, and to carry out their actions to attain the goal of their activities, I call semiotic means of objectification.* (p. 41)

As we can detect from the given quotations, the semiotic means of objectification include: A variety of types of *signs* (words, gestures, inscriptions of different kinds, and so on), and the use of *artifacts* (rulers, calculators, computers and so on).

Different kinds of signs and the use of artifacts can be combined in order to reach what has been called *semiotic nodes*, that are "pieces of the students' semiotic activity, where action, gesture, and word work together to achieve knowledge objectification" (Radford et al., 2003, p. 56). By means of microgenetic analysis in algebraic context and the identification of semiotic nodes, gestures are shown to play a central role in the process of perceptual apprehension of a geometric-numeric pattern and its generalization, especially for their close and complex coordination with other semiotic means of objectification (Radford, Bardini, & Sabena, 2007).

Radford's theorization has strongly influenced the elaboration of the *Semiotic Bundle* notion by Ferdinando Arzarello, as a system made of the different *signs*—and of their *mutual relationships* that are produced by students and possibly teacher in classroom activities. More precisely, a semiotic bundle is defined as:

- ♦ A *set of signs* which may possibly be produced with different actions that have an intentional character, such as uttering, speaking, writing, drawing, gesticulating, handling an artefact.
- ♦ A *set of modes* for producing signs and possibly transforming them; such modes can possibly be rules or algorithms but can also be more flexible action or production modes used by the subject
- ♦ A *set of relationships* among these signs and their meanings embodied in an underlying meaning structure (Arzarello, 2006, p. 8).

In this definition, "sign" is to be intended in an inclusive sense, after Vygotsky's and Peirce's theories. As a matter of fact, in both accounts there is not a limitation upon what can be conceived as sign. Peirce in his representational account of sign defined it essentially as a relationship between a *representamen* (that represents), an *object* (that is represented), and an *interpretant* (another representation referring to the same object, which starts the semiotic process). "A sign, or representamen, is something which stands to somebody for something in some respect or capacity" (C.P., 2.228).

This definition does not make any prescription on what can be considered as a sign: Anything that enters into a process of semiosis can be considered a sign. On the contrary, other semiotic approaches considered in mathematics education, place strong limitation on what can be considered part of the semiotic processes: see for instance Duval's notion of 'semiotic register of representation', which has precise and codifiable rules of productions and transformation (Duval, 2006). Also Peirce's specification of signs in the three main categories of icon, index and symbols seems to point the attention to the complexity of semiotic processes, that has been analyzed deeply also with respect to mathematics epistemology (Otte, 2006).

Also in the Vygotskian's functional perspective on signs, there are not *a priori* constraints on what can be considered as sign, provided that work according to their mediating function at a cognitive level. Signs are tools of reflection that allows individuals to plan action, auxiliary means to organize our behavior.

> *The invention and use of signs as auxiliary means of solving a given psychological problem (to remember, compare something, report, choose an so on) is analogous to the invention of tools in one psychological respect. The signs act as instrument of psychological activity in a manner analogous to the role of a tool in labor* (Vygotsky, 1931/78, p. 52).

In the Semiotic bundle, the notion of *semiotic set* is defined broadening Ernest's definition of semiotic system (Ernest, 2006), with the purpose to include embodied features (and their idiosyncratic modes of production), as well as the classical semiotic registers (e.g. registers *à la* Dual). The word 'set' is used in the mathematical sense of 'collection of tokens'. In Semiotic Bundle studies, the expression 'semiotic resources' is often used as a synonym of the word 'sign', to underline something that is at students' and teacher's disposal in the mathematics classroom. An example can be constituted by students' words, gestures and drawn figures while solving a geometrical problem: all they are different signs that constitute a semiotic set; they are not only present in the same problem-solving activity, some relationships may be found between them. For example, a gesture may give origin to a written diagram, showing a genetic relationship within the semiotic bundle (Sabena, Robutti, Ferrara, & Arzarello, 2012).

From a structural point of view, the Semiotic Bundle is characterized by two key-features.

- Its systemic character, referring to the relationships between the various kinds of signs (which are also called "semiotic sets") at a certain moment (like a sort of "semiotic picture"): *synchronic analysis*.
- And its dynamic nature, focusing the evolutions of signs and of their transformations along the time (a sort of "semiotic movie"): *diachronic analysis*.

Also, in gesture studies we find elements referring to a synchronic analysis, such as the speech-gesture relationship at a certain moment, and to a diachronic analysis, such as the repetition of a certain gesture feature a long time, that is called "catchment" (McNeill, 2005). And, similar to studies in the gesture field, taking the context into account is essential in order to understand the contribution of a single semiotic resource within the semiotic bundle.

Looking at mathematics teaching-learning processes in the classroom within a multimodal perspective has led to identify a new didactic phenomena called the *teacher's semiotic game* (Arzarello et al., 2009). A semiotic game may occur when the teacher interacts with the students, for example in classroom discussions or during the groups' work. In a semiotic game, the teacher tunes with the students' semiotic resources (e.g. words and gestures), and uses them to make the mathematical knowledge evolve towards scientifically shared meanings. For instance, the teacher may repeat a gesture that one student has just done, accompanying it with an appropriate verbal explanation. Semiotic games may contribute to the student's process of appropriation of the culturally shared meaning of signs, provided that the teacher is aware of the importance of considering the multimodality character of the classroom activity.

> [Semiotic games] *allow the teacher to become suitably in tune with students' languages and, conversely, they allow students to achieve resonance with the teacher's languages and, through them, with the*

> *institutional knowledge. In order that such opportunities can be concretely realized, the teacher must be aware of the role that multimodality and semiotic games can play in teaching. Awareness is necessary for designing the conditions that foster positive learning experiences and for adapting her/his intervention techniques to the specific didactic activity.* (Arzarello, Paola, Robutti, & Sabena, 2009, pp. 107-108)

In order to illustrate the semiotic bundle as an analytical lens for multimodal processes in mathematics learning, an example is taken from a classroom teaching experiments carried out in Italy, in kindergarten. The example will be analyzed in detail, with the purpose of providing also a methodological insight on data analysis performed with the Semiotic Bundle tool.

An Example From the Classroom

I will now illustrate the multimodal approach by means of data analysis performed with the Semiotic Bundle lens. Data are taken from video-recordings in classroom context and involve children aged 5 in experimenting with a programmable robot with the guide of their teacher in kindergarten.

The robot is a kind of tridimensional and touchable version of the well-known Logo turtle by Papert (1980). It has a bee-shape and its movement can be programmed through buttons placed on the upper part (Figure 1), that is four arrows for onward and backward steps, right and left turns.

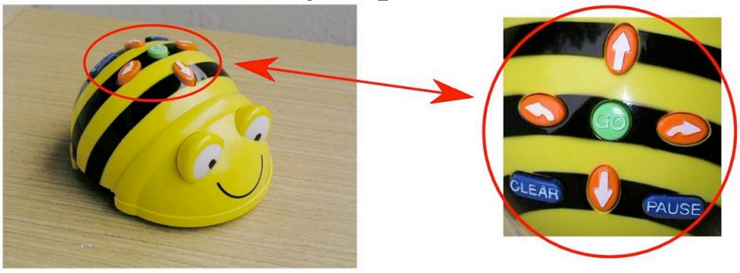


Figure 1. The programmable robot used in the teaching-experiment

The robot bee can move on a plane with 15cm-long steps (the same measure of its length). Steps are marked by a quick stop, which creates a silent pause with respect to the noise of the movement, and by the lightening of its eyes. Pushing the green button "GO", the robot executes the previously programmed sequence. A specific button ("clear") allows the user to clear the memory from past commands, while the button "pause" can add a pause of 1 second.

From a didactical point of view, the activities aimed at promoting competences in spatial thinking and in problem-solving, by involving the children in experimenting with the robot in a playful environment. Extended presentations of the design of the learning activities and discussion of results have been published in (Sabena, 2017). One basic assumption grounding the design is that children since their young age are to be involved both in *meaningful activities*, as well as in *reflective practices* on them, which means setting moments in which children are asked to stop “doing” something and to think about what they did (in their near past) or what they can do (opening the realm of possibilities and not only actualities).

The following example is taken exactly from one of these precious moments, and specifically from a discussion occurring *before* an activity with the bee robot. The activity has been designed by a future primary teacher, who was taking part to the teaching experiment for her Master thesis, in collaboration with the teacher and the author. It is based on a poster placed on the floor and showing a path connecting a house and a flower (Figure 2); the path has been drawn as composed of straight parts as well as of four turns, in a size that can be actually run across by the robot.

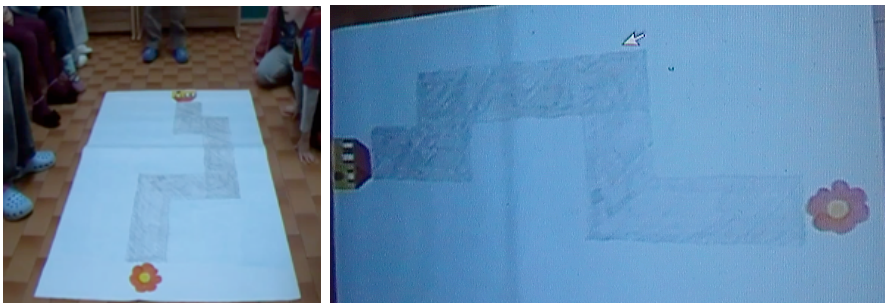


Figure 2. The poster with the path shown to the children

So far, after an initial exploration of the artefact, children have learnt how to send the robot from a place to another in the floor, and have shown their preference in programming only straight movements (without turns). The teacher has set the discussion with the goal to help the children becoming aware of the characteristic of the road along which they will make the bee travel, and in particular of the fact that it is not straight. Let us see how the teacher introduces the discussion and the children begin to answer[1].

1. Teacher:	Today we explore this [looking at the poster]. What comes to your mind looking at this?
2. Stefano:	It is a road.
3. Viviana:	A flower and a house. [Andrea is pointing at the path and moving his hand quickly, doing some noise with his mouth, as Figure 3

[1] The transcript reports on students’ and teacher’s multimodal semiotic resources avoiding technical details (i.e., conventions about overlapping words, tone of voice, and so on are not used). Utterances are translated into English; gestures that are both described and shown in the reported pictures. Names have been changed. The multimodal transcript constitutes therefore a tool of analysis, used together with the video-recording of the lesson, in which all dynamic and prosodic elements can be seized.

shows]

Figure 3. Gestures not accompanied with words: Andrea points at the path and moves his hand quickly, doing some noise with his mouth.

4. Marco: We must arrive at the flower!

5. Giulia: Or the flower must arrive until the house [Anna is silently pointing at the path and slowly moving her hand, as Figure 4 shows].

Figure 4. Gestures not accompanied with words: Anna points at the path and slowly moves her hand.

6. Teacher: And whose is the house?

7. All children: The bees!

8. Paolo: Here, here and here [pointing at the path and moving his hand for three traits (see Figure 5)].

Figure 5. Gestures not accompanied with words: Paolo points at the path and moves his hand for three traits.

9. Teacher: And how is it this road [silently moving horizontally her extended forefinger, as Figure 6 shows]? Is it straight? [Fabio is pointing to the path and moving his hand for five traits, corresponding to the five straight parts of the path (visible in the background in Figure 6)]

10. All children: Noooo![2]

Figure 6. The teacher is extending her forefinger and moving it in horizontal way, while saying “straight”.

After indicating the main elements that characterize the poster, i.e. the road, the flower, and the house (lines 2, 3), the children try to give a sense to the proposed context, in terms of a travel to be performed along the drawn path (lines 4, 5). While some children provide a verbal answer, others are using their extended arm and hand to simulate a movement along the path: the transcript reports the most striking cases of Andrea, Anna, Paolo and Fabio (Figures 3-6). Andrea repeats a quick gesture several times, accompanying it with a noise that could remind the sound made by the bee. On the contrary, Anna keeps silent and appears strongly focused on coordinating her hand movement with the path as she sees it from her position. Paolo, who is seated in a side position with respect to the path, tries first with his left arm, and then with his right one, to indicate some parts in which he seizes the path is composed. His gesture is the only one to be accompanied by words (“here, here and here”), which actually are indicating deictically what is pointed out by the gesture. A similar gesture is performed finally by Fabio, which manages also to identify the five straight traits that constitute the path.

By means of these dynamic gestures, children are *simulating* a motion along the drawn path (possibly they are imagining this motion), as well as they are answering the teacher’s question and *describing* what they see when looking at the poster.

According to the design of the activity, the teacher intends to focus the children’s attention on the turns that appear in the path. In order to single out this specific aspect, she exploits gestures and silence, together with words: in her second question in line 9 first she silently performs a gesture that indicates straight features (moving horizontally her hand keeping her forefinger extended in an evident way, see Figure 6), and then she accompanies the gesture with the word “straight”. By providing an iconic instantiation through her gesture, the

[2] The episode is partly analysed in Sabena (2017). The present analysis further deepens the multimodal aspects.

teacher is making clear the meaning of the word that she intended to get the students' attention on.

We may develop the analysis of the semiotic bundle by means of a table that develops the teacher's and the students' semiotic resources along the time: This table is called "timeline" or "semiotic line" (Sabena et al., 2012). From the example shown in Figure 7, we can notice that the teacher is using the dynamic gesture *after* the students' answers, which were provided through gestures and words. In more general terms, the teacher is tuning with the students' semiotic resources (she is using gestures as well) and is using another resource in order to introduce a new element in the discourse (the word "straight"): This is an example of *semiotic game* of the teacher. Differently from the typical semiotic game as described above, the teacher is not repeating the same gesture as the students just did, but she is providing a *contrasting case*: The students traced short traits one after the other, whereas the teacher is tracing a single long one. In this way, she is asking them to focus on a specific aspect that she considers crucial for the activity. As we will see in the following of the episode, the question prompts the students to provide further clarification.

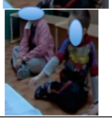
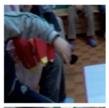
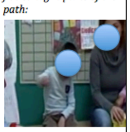
	1. [00:00]	2.	3.	4.	5.	6.	7.	8.	9.	[00:46]
Teacher's words	Today we explore this. What comes to your mind looking at this?					And whose is the house?			And how is it this road *(silently moving horizontally her extended forefinger)*?	Is it straight?
Teacher's gestures										
Children's words		*Stefano*: It is a road	*Viviana*: A flower and a house	*Marco*: We must arrive at the flower!	*Giulia*: Or the flower must arrive until the house		*Chorus*: The bees!	Paolo: Here, here and here		
Children's gestures			*Andrea is pointing at the path and moving his hand quickly*:  		*Anna is pointing at the path and slowly moving her hand:* 			*Paolo is pointing at the path and moving his hand for three traits:*  	*Fabio is pointing to the path and moving his hand for five traits, corresponding to the five straight parts of the path:* 	

Figure 7. The timeline of the reported episode

The timeline is particularly helpful to carry out the semiotic bundle analysis with respect to two dimensions: Looking at different semiotic resources in the same moment (*synchronic analysis*), and looking at relationships between resources in different moments.

As concerns the former, from the timeline in Figure 7 for instance it is immediate to seize that the first teacher's utterance (line 1) is not accompanied by gestures, whereas the second one (line 9) is. In addition, it can be noticed that the children are not apparently listening to each other, because we find all cases in which one student is speaking and another one (indeed more than one) is gesturing without reference to her/his mate's utterance.

As concerns the diachronic analysis, we can see that the line corresponding to the teacher's gestures starts to be filled *after* the student's gestures, and that different students are exploiting the *same* kind of semiotic resource, i.e. dynamic pointing gestures to make sense of the situation and to participate to the shared activity (a case of catchment). Of course diachronic analysis can be better exploited on longer video segments, or as a means to find connections between different episodes within the same teaching-experiment.

The discussion continues with students showing why the road is not "straight".

11. Stefano: It makes curves like this and like this [with his hand he is traveling the road on two consecutive straight parts (Figure 8)]

12. Francesca: Straight is like this [moving her hand on the poster, parallel to one straight part of the road]

Figure 8. Stefano is "travelling" with his hand on the drawn road, before and after a "curve"

Now, asked to describe the road (line 10), children provide a *dynamic description*, apparent in their words, but particularly in their gestures. We see for instance here Stefano, who is travelling the road with his hand (Figure 8) to indicate the "curves" that he mentions; but also other children are pointing to the road moving their hands to imitate turns—see the girl who stands in Figure 8 (left image); other children *touch the entire path with their hands*, without *making any verbal comment* (Figure 9); or Francesca that is showing with her hand the meaning of the word "straight" (line 12).

Figure 9. Children touching the entire drawn road with their open hand, as in a simulated travel

Children's verbal descriptions are quite poor (besides the indication of curves, only deictic terms), and can be understood only considering the co-timed gestures; the teacher uses then a trick to push them to elaborate more elaborated descriptions.

13. Teacher: And then? Let's do like this: I close my eyes and you tell me how is the road, because I do not know it…Is there a starting point? And an arrival? Explain to me.

14. Fabio: The start is in the house and maybe over there—pointing gesture (see Figure 10)—where there is the flower, it is the arrival.

Figure 10. Fabio's pointing gesture indicating the arrival point

15. Teacher: But in this way I would not be able to arrive: you must explain well.

16. Fabio: You must go straight [pointing gesture, as Figure 11 shows (first image)], then turn [moving and turning his body (second and third images), and making a turning gesture with right hand (fourth image)], go still a bit straight, then turn again, go straight and you are arrived at the flower.

Figure 11. Fabio's gestures accompany the introduction of the terms "straight" and "turn". In second, third and fourth pictures also the body rotation can be observed.

17. Teacher: But I don't know where to turn, how can I understand which part to turn… [The children continue to explain mainly with deictic terms such as "here", "there", accompanied by gestures]

18. Teacher: No, no, if you had to explain it only with words?

19. Chiara: Left and right

20. Teacher: Left and right, or towards… So, explain me better, you can do it: not like "I make some curves", but how many, I go straight and how far, or rightwards, or towards the benches, towards the door…

While closing her eyes in order to push the children to provide richer verbal descriptions, the teacher is suggesting them to exploit some *reference points*, such as the starting point and the arrival (line 13). However, even if she is visibly keeping her eyes closed, children continue to use gesture as a main resource accompanying speech, as we can see here Fabio, who stands up and makes a pointing gesture (line 14, Figure 10). After the teacher's prompt (line 15), Fabio refers to an imagined motion along the road and gives a sort of instruction on how to reach the flower starting from the house (line 16). The introduction of two new spatial words, "go straight" and "turn" is accompanied by gestures and body posture. First there is a static pointing gesture made with the extended right forefinger (Figure 11, first image); it points to the initial straight segment of the path, close to the house. Then there is a dynamic gesture made with the full right hand (Figure 11, last image). This gesture is prepared and combined with a body rotation, visible in the sequence of pictures in Figure 12: The body rotation is prepared by a little detachment rightward from the Master student (Figure 11, second and third images), providing the boy with the necessary space to carry out a little leftward rotation.

The body movement and the hand gesture are the only semiotic resources expressing the information about the sense of the rotation (leftwards), which is not made explicit in the verbal utterance. The child concludes then his description using only words, without specifying any quantification for the straight parts, nor the directions of the turns.

The teacher is constantly pushing towards richer verbal descriptions and this goal is finally made explicit to the children (line 18). She gives also a constructive support, by suggesting to quantify the number of steps to be done in each straight part and to refer to *subjective references* (“rightwards”), as well as to *objective ones* (“towards the benches, the door”, line 20). In the subsequent part, children will pick up only the subjective references, and will use the left/right dichotomy to describe the motion along the road. This is coherent with the whole didactical engineering based on the experience of motion.

If at the beginning the students appear not to listen to each other (see comments above based on the timeline in Figure 7), after some minutes of discussion they start coordinating each other. A striking case is shown by the case of Lorenzo and Andrea, facing the following question asked by the teacher:

21. Teacher: Which road has the bee to do from the house to reach the flower?

22. Lorenzo: It goes straight for a while… Then it turns leftwards, then it goes straight for a while. It turns rightwards…and…it arrives at the flower. [Andrea is silently performing the gestures as shown in Figure 12]

Lorenzo's words	It goes straight for a while...	...	Then it turns leftwards, then it goes straight for a while. It turns rightwards	...	and...	it arrives at the flower
Andrea's gestures		*Andrea is placing his hand as parallel to the initial, straight part of the road:* 		*Andrea is pointing to the second turn of the road (a rightward turn)* 		*Andrea is pointing his forefinger in front of him and slowly moving it from left rightwards. He is gazing in front of him.* 

Figure 12. The timeline showing the temporal relationship between Lorenzo’s verbal description and Andrea’s simulating gestures

While Lorenzo is answering verbally, Andrea is silently gesturing: his gestures are performed just after some key elements of Lorenzo’s description and appear coordinating with it. The first one is performed in the air and shows an open hand positioned parallel to the initial part of the road. The second gesture is indicating a specific position in the path that is the position described by Lorenzo’s utterance. And the third one is again in the air, this time not related to the drawn path, nor specifically to Lorenzo’s words, which are not indeed describing the path in a precise way.

Andrea’s gestures appear to simulate the motion of the bee robot along the road and may be interpreted as a strategy used to make sense of his mate’s description—through a mechanism of simulated action, see Hostetter & Alibali (2008). But it is more than this. By means of his gestures Andrea is engaging himself in the shared activity and is trying to face the teacher's question: Gestures allow him to participate to what constitutes the "semiotic symphony" (Radford et al., 2007) that the children are developing.

DISCUSSION

After her research on spatial development in early age from a psychological perspective, Liliane Lurçat (1980) concluded that

> *It seems hard to separate, in the appropriation of the environment realized by the young child, these two sources of knowledge, the one practice, the other verbal, since both converge early in the first months of life* (pp. 15-16, translation by the author).

Reading this sentence after almost forty years, we can recognize the close relationship between *acting* and *communicating*, within is placed by multimodal perspectives at the heart of learning processes in a broader sense. Lurçat's endeavor was indeed oriented in recognizing the role of verbal language since the first developmental steps in a domain that was traditionally considered strictly linked to motor and perceptual experience, that is early spatial conceptualization. Within a multimodal perspective, other kinds of signs are considered besides the verbal ones, in order to include all possible contributions to the semiotic process through which teaching and learning are framed: from written inscriptions, to signs emerging from the use of artifacts, to the embodied dimension.

In the previous section, the Semiotic Bundle tool has been used to analyze classroom excerpts from a teaching experiment in kindergarten: the semiotic resources mainly used in the episodes were verbal utterances, gestures (included body turning and posture), and the written poster with the road. The role of gestures in synergy with the other signs has been analyzed in detail, both at a synchronic and at a diachronic level. The children young age and the spatial context have certainly favored such a flourishing of gestures. However, several studies have shown that gestures play a role also with older students and in other contexts (Edwards, 2010; Radford, 2003; Roth, 2001).

The multimodal semiotic analysis has highlighted two different ways in which children have conceptualized the spatial elements spatial during the artifact-based activities: one is *static* and *global* (static pointing gestures, words indicating objects, counting of angles), while the other is *dynamic* and *paths-based* (dynamic pointing gestures, words of motion, counting of turning). This second one appeared to prevail at the end of the teaching-experiment, as witnessed also by written drawings (Sabena, 2017).

Studies in cognitive science within the embodied mind approach have shown that motion constitutes the source domain of many static concepts. According to the cognitive mechanism called "fictive motion" (Tamly, 2000), static objects acquire meanings in terms of motion. An example common in the mathematics classroom can be that speaking about graphs of functions that are “going up to reach their maximum” or “down at their minimum”.

These results on the one hand provide a cognitive foundation for the whole didactic engineering based on the use of the moving bee-robot, which has fostered the paths-based perspective. Also they can explain why the task of *describing the road* (see lines 1 and 9 in the first excerpt) is soon interpreted by students and teacher as the task of describing *the motion along that road* (see lines 4, 5 and 9, gesture component).

On the other hand, also some distinguished voices from the history of mathematics appear to confirm this perspective. Here we can read how Henri Poincaré (1905) describes the geometrical activity of locating an object in space in terms of motion.

> *Locating an object in any point means representing to himself the motion (that is the muscular sensations that accompany it and that have no geometrical feature), which is necessary to in order to arrive at it*[3] (Poincaré, 1905, p. 67, translation by the author)

As we have seen in the previous section, also teacher’s gestures and embodied resources have been included in the semiotic analysis. This has important didactical implications, if we read the teaching and learning activity in the dialectics between personal and institutional signs, which recalls Vygotsky’s distinction between personal and scientific meanings (Bartolini-Bussi & Mariotti, 2008). In this view, learning consists in students’ personal appropriation of the signs meanings, fostered by social interaction under the coaching of the teacher.

From a functional point of view, gestures can act as personal signs, and the teacher can start a *semiotic game* from students’ gestures in order to support their transition to the scientific meaning. Usually this game starts with the repetition by the teacher of a gesture performed by one or more students (a catchment) plus the simultaneous introduction of a new word. In line 9 of the excerpt we have analyzed a case of semiotic game that is different from previous cases, since the teacher is showing a *contrasting case* with respect to the students’ gestures *and* introducing a new word (‘straight’) in order to focus on a specific aspect that she considers crucial for the subsequent activity but was not present in the students’ signs.

Resonances Between the Semiotic Bundle Analysis and the Theory of Objectification

The multimodal approach presented in this paper has many commonalities with the Theory of Objectification developed by Luis Radford. The two theoretical

[3]Translation into English of the original in French: “…localiser un objet en un point quelconque signifie se représenter le mouvement (c'est-à-dire les sensations musculaires qui les accompagnent et qui n'ont aucun caractère géométrique) qu'il faut faire pour l'atteindre”.

perspectives have been in constant dialogue (Radford & Sabena, 2015; Radford et al., 2017), and positive resonances between them may be found.

Considering a theory as "a way of producing understandings and ways of action based on: a system, P, of basic principles…a methodology, and a set of paradigmatic research questions" (Radford, 2008, p. 320), on the level of principles[4] both approaches have a strong contact point in adopting a Vygotskian perspective on teaching and learning. This entails also a critique of the embodied cognition paradigm, for neglecting the social and cultural dimensions in which mathematical concepts arise and evolve, and the fundamental role of signs and the historical evolution of mathematics therein (Radford et al., 2005).

Also the analysis of signs constitutes a strong commonality, on the levels of both principles and methodology. As a matter of fact, both approaches focus on the evolution of mathematical meanings in students by looking at the evolution of the signs through which the mathematical activity unfolds: the Semiotic Bundle has placed the attention on the relationships between different signs both in specific moments (*synchronic analysis*) and in the evolution of time (*diachronic analysis*), and the Theory of Objectification has elaborated the notion of *semiotic means of objectification* and of *semiotic nodes* in order to frame the various semiotic resources that students and teachers use in order to notice a mathematical structure or concept (Radford, 2009).

Also, in both cases signs are considered in a wide sense, going beyond previous approaches that limited their analysis of specific kinds of semiotic systems—e.g., Duval's notion of semiotic register of representation, see Duval (2006). On a methodological level, data analyses are carried out by means of video-recordings of classroom activities, transcripts, and a micro-analytical stance similar to ethnographical studies (Radford at al., 2017). This practical aspect, made possible by technological development of the last two decades, has certainly constituted a catalyst to the emergence of gestures as a new focus of attention in mathematics learning: as in hard sciences, a technical aspect of research has made possible the actual observation of new phenomena, and as a consequence new theorization has been elaborated.

On the other hand, the analysis of gestures has been nurtured also by the growing bulk of results coming from other disciplines, in particular by the psychological domain. The influence of such studies is particularly evident in the Semiotic Bundle approach (see for instance the embedment of the notion of catchment within the diachronic analysis), whereas the Theory of Objectification has more deeply referred to philosophical stances. In summary, both theories have been strongly influenced by two sources: the empirical investigation from

[4]With reference to such distinction, the Semiotic Bundle can be considered a methodological tool of the theory. For an extended discussion of this theoretical aspects, see Arzarello & Sabena (2014).

the classroom, enhanced by video recordings, and results from disciplines outsides the mathematics education field.

Furthermore, in both approaches the classroom context is much more than an empirical data collector, and for two reasons. The first regards an *interpretative view* on learning processes. This stance is shared by most research studies in a post-modern perspective. As concerns the Semiotic Bundle tool, I stress that even the most precise timeline (e.g., Figures 7 and 12) is already the result of the researcher's selection (pictures to be inserted, organization of the lines, for instance) and so of an interpretative work. Second, the theoretical perspectives are not only used to analyze data *a posteriori*, but also to inform the *classroom activities design*:

> *We do not register the educational phenomenon in order to offer plausible interpretations of it. Although we provide interpretations, we also design the classroom activities, and by designing them, we alter and transform the manners in which teaching and learning co-occur.* (Radford & Sabena, 2015, p. 979)

In the Italian mathematics education research tradition, this aspect is usually referred to as 'research for innovation in the classroom' (Arzarello & Bartolini Bussi, 1998), which shares many contact points with the so-called design-based research (DBRC, 2003).

With respect to the design dimension, the multimodal approach provides a general frame that privileges the social dimension, though which signs are used to communicate between students and with the teacher, and the reference to cognitive aspects of mathematical learning, without neglecting the cultural ones (see the discussion above on motion as a source of mathematical knowledge).

Within this frame, the role of the *teacher* soon emerged as an important element of analysis. The Semiotic Bundle analysis has led to identify the phenomenon of the semiotic game of the teacher, of which we have seen an example in this paper (specifically, a *contrasting* case).

Research on the conditions in which semiotic games are successful with respect to students' learning is still underdeveloped and appears to require going beyond the theoretical elements of the multimodal perspective as presented in this paper. It requires to face openly the problem of what constitutes 'knowing' and the how the processes of 'getting to know' develop in the students, that is to say in the human beings: an aspect that has worked as a powerful motor for the development of the Theory of Objectification. On the contrary, this aspect is not theorized in the Semiotic Bundle notion. As a methodological tool, such a notion may be easily coordinated or local integrated with different theoretical perspectives (e.g. constructivist theories), provided they take into account the multimodality of teaching and learning (see the book edited by Bikner-Ahsbahs and Prediger in 2014, for a discussion on meta-theoretical aspects and examples).

On the other hand, when the role of the teacher is to be framed more closely, such a theoretical space needs to be filled, and adopting a phenomenological perspective appears as a promising road (Arzarello et al., 2011). Possibly, future development of the Theory of Objectification will provide useful elements also in this crucial direction and the dialogue with the Semiotic Bundle approach will benefit from them.

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