The Mathematics Teacher and Curriculum Change

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One of the most compelling lessons of the new math reform era —from the mid 1950s to the mid 1970s— concerned the pivotal role of the mathematics teacher in effecting curriculum change. Recent efforts to change the school mathematics curriculum are rediscovering that old lesson: The teacher is the key to change. Consequently, when teachers are confronted with arguments against the direction that proposals for curriculum change are taking, it is important for them to analyze and discuss the proposed changes. Recent experiences in the United States, which does not have the same centralized curriculum organization as Portugal but which is experiencing some of the same proposals and arguments against them, may be helpful in understanding the role that teachers can play in the social process of creating a curriculum.

Keywords: Curriculum reform; Math wars; Mathematical proficiency; School algebra; Standards; Teacher's role

El Profesor de Matemáticas y el Cambio de Currículo

Una de las lecciones más destacadas extraídas de la era de la reforma de la matemática moderna hacía referencia al papel esencial que el profesor de matemáticas desempeñaba en el cambio del currículo. Los recientes esfuerzos por cambiar el currículo de las matemáticas escolares están redescubriendo esta vieja lección: El profesor es la clave para el cambio. Consecuentemente, cuando los profesores argumentan en contra de las propuestas de cambio curricular, es importante que analicen y discutan los cambios propuestos. Las experiencias recientes en Estados Unidos, donde no existe la misma organización curricular centralizada de Portugal pero que experimenta parte de las mismas propuestas y argumentos contra ellas, puede ser útil para comprender el papel que los profesores pueden jugar en el proceso social de creación de un currículo.

Términos clave: Álgebra escolar; Competencia matemática; Estándares; Math wars; Papel del profesor; Reforma del currículo

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The idea that the school curriculum is something to be changed in a systematic way was one of the twentieth century's contributions to education. Curriculum development projects began to emerge in the mid 1950s as countries sought to catch the "new math" wave that was sweeping across Western Europe and North America (Howson, Keitel, & Kilpatrick, 1981). The term *new math* "was a label not so much for a cohesive set of reform proposals and activities as for an era during which a variety of reforms were undertaken" (Stanic & Kilpatrick, 1992, p. 413).

From the 1970s to the mid 1980s, Portugal's school mathematics curriculum had "a strong 'new math' flavor" (Ponte, Matos, Guimarães, Leal, & Canavarro, 1994, p. 349), but that began to change after 1986, when the Ministry of Education instituted a reform of the system of public education. The case study by Ponte et al. of a pilot program of a new mathematics curriculum in 1990-1991 showed how much its implementation depended on the views and attitudes of the teachers as well as the students. Portugal's experience in the early 1990s appears to have been much like that of other countries when they were adopting new math reforms. Regardless of the nature of a project to change the school mathematics curriculum, it appears that the role of the teacher is critical. "Every teacher is involved in curriculum development, whatever curriculum he [or she] follows, and there are obvious reasons why he [or she] should know as much as possible about its construction and be able to examine it critically." (Howson et al., 1981, p. 259)

Views of Curriculum Change

When I spoke at the seventh conference on mathematics education research in Mirandela in 1998, I said the following:

> *I begin with a famous saying in American political life: "All politics is local. Don't forget it." This saying is usually attributed to Tip O'Neill, who was for many years the Speaker of the U.S. House of Representatives. Actually, however, O'Neill attributed the saying to his father, who said it to him when O'Neill lost his first election, to the Cambridge, Massachusetts, city council. He wanted to remind his son that politics requires that you begin with the people who will vote for you —you need to ask for their vote and to give them a reason to vote for you by saying what you will do if elected (O'Neill, 1994). I have adopted this saying to the question of curriculum. I claim: All curriculum change is local, and personal.* (Kilpatrick, 1998, 1999)

In that presentation I pointed out that the school mathematics curriculum can be seen from several angles: As a set of experiences designed to promote the learning of mathematics and as the course that students follow. When people attempt to locate the curriculum, they often borrow the familiar three levels of curriculum used in the Second International Mathematics Study (SIMS):

- *Intended*. The administrator's point of view.
- *Implemented.* The teacher's point of view.
- *Attained or realized.* The student's point of view.

This way of splitting the curriculum according to the perspective of the participants continues to have some analytic value. In particular, it seems to be useful for comparative studies like SIMS, the Trends in International Mathematics and Science Studies (TIMSS), and the Programme for International Student Assessment (PISA) that make use of questionnaires and tests.

The three-level approach, however, requires that one make the questionable assumption that curricular power flows directly from administrator to teacher to student. It narrows the view of the education process: Whose intentions are represented in the intended curriculum? What about teachers' intentions? Students' intentions? Is there only one intended curriculum? The approach casts the teacher as an obedient employee who is given a curriculum to implement and who plays no role in co-constructing the curriculum along with students. It offers a top-down view of the curriculum and therefore of change.

In my view, the intended curriculum is not a curriculum itself. Instead, it is a blueprint for a curriculum to be realized. The word *curriculum* comes from the Latin word for course or career. It refers to actual experience; it is not about intentions, but reality. In the analogy I used in 1998: The intended curriculum is to the real curriculum as the architect's plan is to the building.

Also in 1998, I pointed out that education is a complex of nested systems, beginning at the outside with the national educational system and ending in the classroom system, nested within all the others. From the classroom out to the country, each system has structural units aligned, at the national and local levels, with political units. The mathematics being taught and learned is located within the classroom, although each system that encompasses the classroom has a view of what that mathematics should be. In that sense, we can view the curriculum as a hierarchy —decisions at the top filter down to classrooms—. But more accurately, these educational systems are interlocking and interpenetrating. The vector of change efforts can begin with any one of them and proceed to any other. The analogy I prefer, and one that suits well the seagoing history of Portugal, is that the curriculum is like the *ocean*. At the top, where the nation talks about its mathematics curriculum, change may seem obvious. But on the ocean floor, where the curriculum lives, life is different. The movements above may or may not affect what happens there.

Curriculum systems have what Ian Westbury (1980) terms a *deep structure* that requires the recognition that many of the elements of the surface structure of those systems (for example, syllabi, guidelines, given kinds of examination,…) may have, in particular cases, a problematic influencing relationship on both the curriculum as it is found and attempts to modify or change that curriculum. (pp. 15-16)

One should not assume that strong centralized control of the curriculum will enable curriculum change to come from the top down. Despite claims to the contrary, school systems are very much alike. I repeat one of my favorite quotations, which remains true today in most countries even though England now has a national curriculum:

> *Centralised systems are not so centralised and decentralised systems are not so decentralised, as commonly supposed. As a French school inspector once observed: "In France, every teacher is supposed to be doing the same thing but nobody is, and in England, where everyone is supposed to be going his own way, nobody is."* (Howson et al., 1981, p. 58)

Although the surface of the curricular ocean may sometimes appear to have been swept by a tsunami, therefore, down in the depths curricular life goes on unperturbed. For example, one can argue that every wave of curriculum reform in U.S. school mathematics ("unified mathematics," "new math," and "standards") has left the curriculum unreformed. Changed, perhaps, but not reformed. And the changes that these reform efforts did bring about were not the changes the reformers intended. The strong rhetoric of reform has actually been a mask for disunity, contradiction, misinterpretation, and indifference. A reasonable estimate might be that for the reform efforts in the U.S., fewer than 10% of mathematics teachers have been professionally involved in curriculum change. Nonetheless, the public is very much aware of reactions to reform efforts even if they are not necessarily well informed about the substance of the reforms.

Math Wars

As Isaac Newton observed, for every action, there is an equal and opposite reaction. Certainly it has been the case in the United States that each 20th-century attempt to reform the school mathematics curriculum met with an effort at counter-reform. During the new math era of the mid 1950s to mid 1970s, the term "math wars" was introduced to describe the conflict between reformers and their critics (DeMott, 1962). More recently, reform efforts have begun again, this time spurred by efforts of the National Council of Teachers of Mathematics (NCTM). In 1980, NCTM published its document *Agenda for Action*, which proposed that problem solving be the focus of school mathematics and that basic skills be redefined to extend beyond mere computation. The *Agenda* provided direction for reform; it was essentially NCTM's first effort to influence national educational policy in a substantive way. The Council began to hold meetings to make recommendations on how school mathematics should change, and it formed committees to help teachers select textbooks and evaluate programs.

Standards Documents

In 1984, NCTM formed a taskforce to formulate guidelines for the school mathematics program for Grades Kindergarten to 12, which led to the publication in 1989 of *Curriculum and Evaluation Standards for School Mathematics*. That publication was followed in 1991 by standards for teaching and in 1995 by standards for assessment (NCTM, 1991, 1995). And the standards for curriculum and evaluation also prompted standards in other school subjects.

When the standards were first proposed, the reaction was generally positive. For example, textbook publishers began to label their books "standards based." The National Science Foundation funded projects to develop new instructional materials for the middle school and high school grades. District and state curriculum standards began to be aligned with the NCTM standards, and politicians praised the NCTM for its leadership role in improving the curriculum. But gradually a backlash began to set it. Columnists and editorial writers began to complain about standards efforts, using terms like "fuzzy math," "whole math," and the "new-new math." Standards supporters answered back, calling current school mathematics "parrot math." Parents and some mathematicians told anecdotes of students failing to learn "basic facts."

Eventually, the U.S. Secretary of Education, Richard Riley, speaking at a joint meeting of the American Mathematics Society and the Mathematical Association of America in January 1998, called for a cease fire in the "math wars," which was one of the first times that the term was used after the new math era. It may be useful to take a brief look at the way the recent math wars differ from the earlier ones. First, the reform efforts during the new math era were initiated primarily by mathematicians, whereas the standards-based reforms were initiated primarily by a teachers' organization, NCTM. Second, the stimulus for reform during the new math era was both the perceived gulf between school and university mathematics and the political and military threat posed by the Soviet Union. In contrast, the standards-based reform efforts were stimulated by international comparative studies of mathematics performance, such as those mentioned above (SIMS, TIMSS, and PISA), as well as by concerns about U.S. economic and technological competitiveness with countries, particularly Asian countries, whose students performed especially well in such studies. Third, during the new math, the movement was primarily opposed by teachers —mostly elementary school teachers—, some parents, and some mathematicians —principally applied mathematicians—, but the general public and politicians were not much involved in the dispute. The recent math wars, in contrast, have been the subject of much media discussion, and opposition —in addition to parents and some teachers— has included quite a few mathematicians as well as politicians they have influenced. Fourth, the substance of the reform effort has been quite different. During the new math, the proposed changes involved the subject matter taught. Efforts were made to acquaint students with the abstract structures of mathematics so that they would better understand what school mathematics was about. Today, the reforms are more pedagogical in intent. Although some reformers aim at a greater inclusion of topics from applied mathematics in the curriculum, most efforts are aimed at getting students more actively involved in learning the mathematics by making the content more meaningful and engaging them in investigative work.

Adding It Up

One response to the recent math wars was the formation, by the National Research Council (NRC), of a committee to conduct a mathematics learning study. An earlier NRC committee had conducted a study on the prevention of reading difficulties and had dealt with the dispute in the reading community between those favoring what was called a "whole language" approach and others who wanted to emphasize phonics in reading instruction. Their report was seen as putting an end to the "reading wars," and so the NRC thought that another committee might end the math wars.

In 1998, the U.S. Department of Education and the National Science Foundation asked the National Academy of Sciences to establish a committee to conduct a study on mathematics learning. After 2 years of work, the committee released its 480-page report entitled *Adding It Up* (NRC, 2001). Then, to disseminate the results to a wider audience, the NRC produced a 52-page version (Kilpatrick & Swafford, 2002), a copy of which was sent to the superintendent of every school district in the United States.

The goals of the mathematics learning study were to make recommendations for improving student learning of mathematics in prekindergarten through Grade 8. Specifically, the charge to the committee listed three goals:

- ♦ To synthesize the rich and diverse research on prekindergarten through eighth-grade mathematics learning.
- ♦ To provide research-based recommendations for teaching, teacher education, and curriculum for improving student learning and to identify areas where research is needed.
- ♦ To give advice and guidance to educators, researchers, publishers, policy makers, and parents. (NRC, 2001, p. 26)

The committee undertaking the study had 16 members with expertise in classroom practice, the mathematical sciences, research in cognitive psychology, business, and research in mathematics education.

Early in their deliberations, the committee decided to focus the report on the topic of number in the curriculum. Number is at the heart of prekindergarten to Grade 8 mathematics, is central to many of the controversies over mathematics teaching, is the most thoroughly investigated part of the mathematics curriculum, leads to algebra, and is strongly connected with other parts of the school mathematics curriculum. In the final analysis, however, the choice was pragmatic. The committee did not have time or resources to investigate the research on all parts of the curriculum —although they did end up addressing some of the other parts—.

One issue for the committee concerned the language to be used in describing the goals of mathematics learning. They considered the terms *literacy*, *numeracy*, *mastery*, and *competence* but rejected each of those terms, primarily because of other connotations, some of them negative, that each term had. The term the committee finally chose was *proficiency*. Because the math wars had often resulted in simplistic claims that pitted skill against understanding, with some people saying that understanding needed to precede skill, follow skill, or replace skill in the curriculum, the committee decided to formulate proficiency so that it would include not only skill and understanding but other qualities as well. The metaphor of a rope woven of strands was adopted, with the five strands to be developed in concert and not one before or after the other. The rope model could then be used to define learning goals for all students, and a variation of the model could be used to discuss proficiency in teaching. The committee also used the five-strand model to organize their discussion of research —how the research on a topic could be synthesized and where it fell short of providing assistance—.

The five strands of mathematical proficiency are as follows (see Figure 1):

- *Conceptual understanding*. Comprehension of mathematical concepts, operations, and relations.
- *Procedural fluency*. Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- *Strategic competence*. Ability to formulate, represent, and solve mathematical problems.
- *Adaptive reasoning*. Capacity for logical thought, reflection, explanation, and justification.
- *Productive disposition*. Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

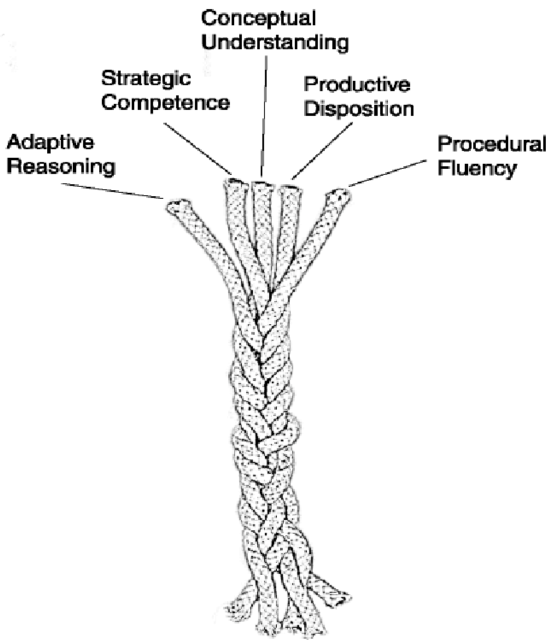

Figure 1. The five strands of mathematical proficiency (NRC, 2001, p. 117)

These five strands of mathematical proficiency have come to be used in formulations of goals and standards for the curriculum and are sometimes being used in framing instructional materials. The five strands are quite similar to the conceptual framework for mathematics used in Singapore (see Figure 2), even though the two formulations were constructed independently.

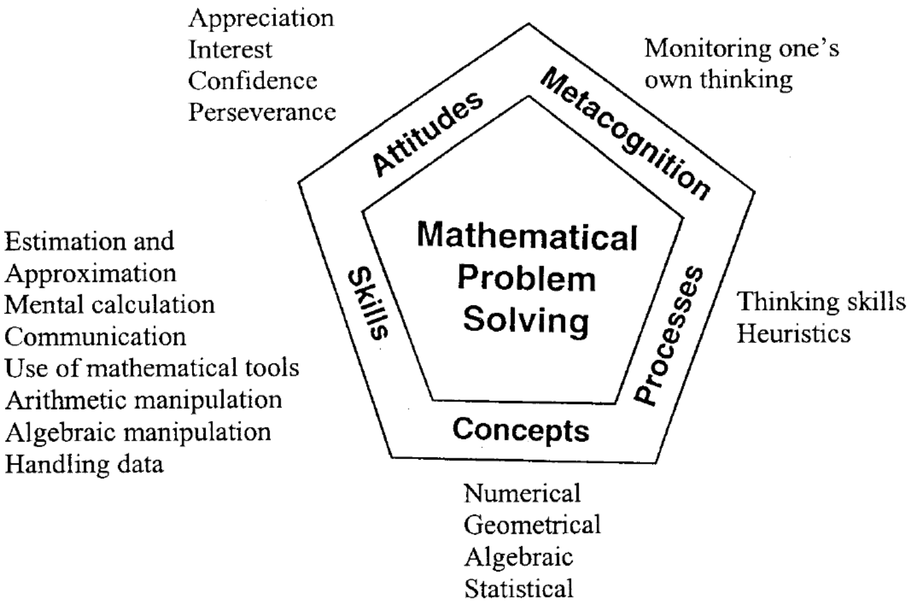

Figure 2. The Singapore mathematics framework (see, e.g., Ginsburg, Leinwand, Anstrom, & Pollock, 2005, p. 15)

A recent tribute to the *Adding It Up* (NRC, 2001) publication was given by Keith Devlin (2008) in his column in the *MAA Online* newsletter and Web site. Discussing the knowledge and understanding of arithmetic that students should have when they leave secondary school, Devlin says that expectation is stated clearly, and up front, in what is generally regarded as the "Bible" of K-8 mathematics education in the U.S., namely the book *Adding It Up: Helping Children Learn Mathematics*, authored by the Mathematics Learning Study Committee of the National Research Council, and published by the National Academies Press in 2001. (It's a great resource that every math teacher and every homeschooling parent should read and consult regularly.)

Curriculum Focal Points and Lenses

In part to address the complaints against the U.S. school mathematics curriculum that characterized it as "a mile wide and an inch deep" (Schmidt, McKnight, & Raizen, 1997, p. 2), and in part to address the way in which NCTM standards documents (particularly NCTM, 2000) were organized by grade-level "bands" (Grades Pre-K to 2, 3 to 5, 6 to 8, and 9 to 12) and not by grade level, the NCTM undertook a project to identify a small number of focal points for emphasis at each grade from prekindergarten to Grade 8. In general, teachers and other mathematics educators saw the resulting report, *Curriculum Focal Points* (NCTM, 2006), as a positive contribution to the discussion of standards. Even though the NCTM made strong efforts to inform reporters about the purpose of the document —to bring focus and coherence to school mathematics— it was portrayed in the media as a concession by NCTM to its critics in the math wars conflict. For example, the *Wall Street Journal*, on its front page, began an article titled "Arithmetic Problems" with the following sentence: "The nation's math teachers, on the front lines of a 17-year curriculum war, are getting some new marching orders: Make sure students learn the basics" (Hechinger, 2006, p. A1). The article went on to quote Ralph Raimi, a mathematician at the University of Rochester, as saying that NCTM's "new guidelines constitute 'a remarkable reversal, and it's about time.'" Eventually, NCTM decided that there was some benefit if people thought the math wars were over.

NCTM undertook another project in 2006 to develop a framework to guide future work in high school mathematics. The committee overseeing the project has produced a first document (NCTM, 2008), which departs from the strategy used in the *Focal Points* document. Instead of giving a small number of focal points for each grade, it takes a broad theme "reasoning and sense making" and addresses how that theme might be manifested in the high school mathematics curriculum. The document has been referred to as the "lenses" document even though the term lens does not appear in the title and is used only once in the document to refer to the way reasoning and sense making might be seen. Reactions to the document are currently being solicited by NCTM on its Web site.

Foundations for Success

On 18 April 2006, President George W. Bush issued an executive order creating a National Mathematics Advisory Panel to advise him and Secretary of Education Margaret Spellings on the best use of scientifically based research on the teaching and learning of mathematics. The panel was asked to make recommendations, based on the best available scientific evidence, on the following:

- the critical skills and skill progressions for students to acquire competence in algebra and readiness for higher levels of mathematics;
- the role and appropriate design of standards and assessment in promoting mathematical competence;
- the processes by which students of various abilities and backgrounds learn mathematics;
- instructional practices, programs, and materials that are effective for improving mathematics learning;
- the training, selection, placement, and professional development of teachers of mathematics in order to enhance students' learning of mathematics;
- the role and appropriate design of systems for delivering instruction in mathematics that combine the different elements of learning processes, curricula, instruction, teacher training and support, and standards, assessments, and accountability;
- needs for research in support of mathematics education;
- ideas for strengthening capabilities to teach children and youth basic mathematics, geometry, algebra, and calculus and other mathematical disciplines;
- such other matters relating to mathematics education as the Panel deems appropriate; and
- such other matters relating to mathematics education as the Secretary may require. (National Mathematics Advisory Panel [NMAP], 2008, pp. 71-72)

The panel, chaired by a former president of the University of Texas, contained 19 members, with 5 ex-officio members from federal government agencies. To accomplish their work, which required 2 years, they formed five task groups and three subcommittees. The task groups were on conceptual knowledge and skills, learning processes, teachers and teacher education, instructional practices, and assessment. The subcommittees were on standards of evidence, instructional materials, and a national survey of Algebra I teachers. The panel’s final report, *Foundations for Success*, was released on 13 March 2008. An electronic version of 120 pages is available on the U.S. Department of Education Web site; a hard copy version of 90 pages can also be ordered there. The task group and subcommittee reports are available separately on the Web site or can be ordered as a single document of 870 pages.

In general, the report received a favorable response in the mainstream electronic and print media, especially for its focus on algebra. In contrast, some mathematics teachers and other mathematics educators saw the focus on algebra as part of the problem. They saw the report as oversimplified and incoherent with respect to the school curriculum, putting too much emphasis on arithmetic, and having an out-of-date view of algebra. Also, there was much criticism of the report of the subcommittee on standards of evidence. The definition of "best scientific evidence" was seen as too narrow and as ruling out much useful research.

There is no explicit definition of *algebra* in the report. Instead, the panel discusses what it calls school algebra:

> *School algebra is a term chosen to encompass the full body of algebraic material that the Panel expects to be covered through high school, regardless of its organization into courses and levels. The Panel expects students to be able to proceed successfully at least through the content of Algebra II.* (NMAP, 2008, p. xvii)

The report then provides a list of major topics of school algebra (see Figure 3).

Symbols and Expressions
- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

Linear Equations
- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

Quadratic Equations
- Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions
- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions

Figure 3. The major topics of school algebra (NMAP, 2008, p. 16)

- Trigonometric functions
- Fitting simple mathematical models to data

Algebra of Polynomials

- Roots and factorization of polynomials
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's Triangle)

Figure 3 (continued). The major topics of school algebra (NMAP, 2008, p. 16)

An interesting contrast can be seen in the inconsistent way the report handles research evidence. Addressing what they call "benchmarks for critical foundations for algebra," the panel found no empirical research on benchmarks or placement of topics that met their criteria for quality. Nonetheless, they set benchmarks, arguing that the benchmarks were justified "in a comparison of national and international curricula" (NMAP, 2008, p. 19). In contrast, addressing the question of integrated curricula, where high-achieving countries do not have the same organization as the United States does —in those countries, topics such as algebra, geometry, and trigonometry are not treated separately in yearlong or half-yearlong blocks— the panel argued instead that there is "no basis in research for preferring one or the other" (NMAP, 2008, p. 22).

The report has been criticized by members of the statistics community in the United States because it makes no mention of statistics. The topics of geometry and measurement are not treated as full topics in their own right; instead, they are treated as adjuncts to algebra. And the report makes reference to "standard algorithms" of arithmetic even though a number of different algorithms are used in U.S. curriculum materials.

I am interested to know how Portuguese mathematics and mathematics educators have seen the report. I have heard that some mathematicians and editorial writers have used the report to argue against, for instance, what they call the "devaluation of calculation," "focus on learning mathematics with understanding," "too early use of technology," and "emphasis on problem solving in mathematics teaching." I do not see the report as making good arguments against these so-called reform proposals, so I would be interested to hear more about the Portuguese teachers' response.

Teachers Creating a Curriculum

Lester Maddox, who served as governor of the state of Georgia from 1967 to 1971, was once asked what could be done to reduce the number of riots in Georgia prisons. He gave a classic response: "What we need is a better class of prisoner."

In a similar fashion, Donald Rumsfeld, who was the U.S. Secretary of Defense from 2001 to 2006, was asked in December 2004 by an Iraq-bound soldier in Kuwait why the soldiers did not have sufficient armor for their combat-bound vehicles. Rumsfeld gave another classic response: "You go to war with the army you have, not the army you might want or wish to have at a later time."

Both responses deny the truism that, as Cassius says in *Julius Caesar*, "the fault, dear Brutus, is not in our stars, but in ourselves." Policymakers tend to shift attention away from their own failings and put it on circumstances outside their control.

After the U.S. elections in 1994 showed that candidates could be elected by working on the national scene rather than locally, some commentators claimed that Tip O'Neill's maxim was no longer valid: All politics is *not* local, they said. Might it also be that all curriculum change is not local? After all, we now have new communications media and much easier access to information. Perhaps the new technologies might change how we think about curriculum change. I think not, however. Change requires much more than information.

Again, as I noted in 1998, a common complaint one still hears from the American public is the following: "We can put a man on the moon; why can't we solve educational problems?" The implication is that we simply lack the technical knowledge needed to solve such problems. Shortly after the first moon landing, however, Irving Kristol (1973) made the following point: Putting a man on the moon is nothing but a technological problem, whereas improving education is anything but a technological problem. Social change involves changing people, and you cannot update people as though they were software.

We need to understand that curriculum change is not a technical matter. Instead, it is a personal journey for mathematics teachers. Any attempts to change the curriculum —including the decision by the Portuguese ministry of education to begin implementing your new school mathematics curriculum next year— need to take teachers where they are and invite them to join the process of reflection and mutual encouragement. Politicians and policymakers also need to understand when teachers do not join the process enthusiastically and instead decide to respond in their own way. We all should recognize that ultimately much of the curriculum —and of curriculum change— lies beyond the domain of educational research or policy.

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