DESIGNING PROFESSIONAL LEARNING TASKS FOR MATHEMATICS LEARNING TRAJECTORIES

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In this paper, we present an emerging set of learning conjectures and design principles to be used in the development of professional learning tasks that support elementary teachers' learning of mathematics learning trajectories. We outline our theoretical perspective on teacher knowledge of learning trajectories, review the literature concerning mathematics professional learning tasks, offer a set of initial conjectures about teacher learning of learning trajectories, and articulate a set of principles to guide the design of tasks. We conclude with an example of one learning trajectory professional learning task taken from our current research project.

Keywords: Learning trajectories; Professional development; Teacher learning

Diseño de tareas de aprendizaje profesional para trayectorias de aprendizaje de matemáticas

En este artículo, presentamos un conjunto emergente de conjeturas de aprendizaje y de principios de diseño para ser empleados en el desarrollo de tareas de aprendizaje profesional que apoyan el aprendizaje de trayectorias de aprendizaje de matemáticas de maestros de primaria. Describimos brevemente nuestra perspectiva teórica sobre el conocimiento del profesor acerca de trayectorias de aprendizaje; revisamos la literatura sobre tareas de aprendizaje profesional, presentamos un conjunto de conjeturas iniciales acerca del aprendizaje del profesor sobre trayectorias de aprendizaje; y articulamos un conjunto de principios para guiar el diseño de tareas. Concluimos con un ejemplo de una tarea de aprendizaje profesional que ha sido tomada de nuestro proyecto de investigación actual.

Términos clave: Aprendizaje del profesor; Desarrollo profesional; Trayectorias de aprendizaje

In recent years, researchers have extended Simon's (1995) hypothetical learning trajectory construct to include empirically-supported descriptions of the ways in which student thinking evolves over time. Based on syntheses of the research literature, clinical

Wilson, P. H., Sztajn, P., & Edgington, C. (2013). Designing professional learning tasks for mathematics learning trajectories. *PNA, 7*(4), 135-143.

interviews, teaching experiments, and large-scale assessment data, learning trajectories (LTs) have come to represent empirically defined descriptions that trace the ways in which students' informal ideas mature through appropriate instructional opportunities into sophisticated mathematical understandings (Confrey, Maloney, Nguyen, Mojica, & Myers, 2009). In the United States, LTs are purported to be a "tool for reform" (Corcoran, Mosher, & Rogat, 2009) and are growing in their influence over national standards development, assessment systems design, and mathematics curricula development.

Early accounts suggest that teachers' knowledge of an LT improved their own mathematics content knowledge (Mojica, 2010), guided their instructional decisions (Wilson, 2009), and enhanced their abilities to use student thinking (Clements, Sarama, Spitler, Lange, & Wolfe, 2011). Though there is a call to the research community to "translate the available LTs into tools for teachers" (Daro, Mosher, & Corcoran, 2011), research on teacher learning of LTs is only beginning to emerge. Empirical work is needed to examine not only the ways in which teachers come to learn about these trajectories but also to define the ways in which teacher educators can design professional learning tasks (PLTs) that support such learning.

The goal of this paper is to present an emerging set of learning conjectures and design principles to be used in the development of PLTs that support elementary teachers' learning of LTs. As part of a larger design experiment to examine a professional development setting in which elementary teachers learn about one particular learning trajectory, this paper highlights the design aspect of the empirical work under way in the project. We begin with background information about LTs and outline our theoretical perspective on teacher knowledge of LTs. Next, we review the existing literature concerning the mathematics PLTs. In the tradition of design research, we offer a set of initial conjectures about teacher learning of LTs and articulate a set of principles to guide the design of PLTs for mathematics LTs. We conclude with an example of one LT PLT taken from our current research project to illustrate the ways that our conjectures and principles may be instantiated.

BACKGROUND

LTs have been defined as

> *descriptions of children's thinking and learning in a specific mathematical domain, and a related conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking.* (Clements & Sarama, 2004, p. 83)

More recently, Confrey and her colleagues defined an LT as "a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time" (Confrey et al., 2009, p. 347). This definition establishes the impossibility of separating student learning from instruction in school settings; it centralizes the fundamental role that mathematics teachers play in the growth of students' understanding of mathematics.

Much of the research in developing LTs makes a distinction between the logic of the discipline and the logic of the learner or the learners' cognitive development (Corcoran et al., 2009). This distinction indicates that, rather than organizing mathematical topics and learning experiences for children based on logical analysis of disciplinary knowledge, LTs allow mathematics instruction to be based on "research about how students' learning actually progresses" (p. 8). This distinction shifts the organizing focus of mathematical instruction from the discipline to the students. Thus, a fundamental characteristic of LTs is attention to the ways a learner's logic matures into the logic of the discipline.

THEORETICAL PERSPECTIVES

In our work, we consider Ball, Thames, and Phelps' (2008) notion of mathematical knowledge for teaching (MKT) in light of the distinction between the logic of the discipline and the logic of the learner. Ball and colleagues built on Shulman's (1986) work on understanding teacher knowledge to conceptualize MKT grounded in an examination of the mathematical knowledge teachers need for teaching. At the heart of their MKT framework was a careful analysis of the mathematical demands teachers face in practice. Their work resulted in "refinements to the popular concept of pedagogical content knowledge and to the broader concept of content knowledge for teaching" (Ball et al., 2008, p. 390).

MKT is organized as two large domains: pedagogical content knowledge (PCK) and subject matter knowledge (SMK). Each of these domains is further divided into three categories of teacher knowledge. Within the PCK domain of the MKT framework, knowledge of content and students is defined as the "knowledge that combines knowing about students and knowing about mathematics" (p. 401) so that teachers may anticipate what students are likely to think as well as what they find confusing, interesting, or motivating. Knowledge of content and teaching refers to knowledge about the design of instruction for a particular content, including choosing examples, sequencing tasks, and evaluating advantages and disadvantages of various representations, in ways that bring together mathematical understanding and an understanding of the pedagogical choices that affect student learning. Finally, knowledge of content and curriculum is placed as part of PCK.

Within SMK, Ball and colleagues explained that common content knowledge is the knowledge of mathematics not specific to teaching, whereas specialized content knowledge is the mathematical knowledge not typically needed for purposes other than teaching. This specialized knowledge is exemplified as the knowledge teachers need to explain patterns in student errors or decide whether a nonstandard approach would work in general. A third category, horizon content knowledge, represents "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 403).

Although we recognize the importance of examining the knowledge demands of teaching and the contribution Ball and colleagues made in defining MKT, our work on LTs focuses on the careful attention to the logic of the learner and the analysis of the relation between this logic and the mathematical disciplinary knowledge. This attention necessitates an interpretation of the MKT categories in light of the relation between the learner and the discipline, particularly when trying to understand teacher learning about LTs. From an LT perspective, we consider the PCK domain as related to the knowledge that emerges from a focus on the learner's cognitive development and is based on teachers' understandings of the learner's logic and how it progresses over time. The SMK domain represents aspects of teacher knowledge that are centred on the logic of the discipline and allow teachers to situate the logic of the learner within the larger framework of shared mathematical knowledge. Together, teachers' PCK and SMK guide the instructional work needed to support students' movement along various levels of an LT as informal ideas develop into sophisticated mathematical knowledge. Therefore, we contend that teachers' knowledge of LTs spans both the PCK and the SMK domains of MKT, and teachers' learning of LTs impact both these domains.

Teacher Learning and Professional Learning Tasks

Silverman and Thompson (2008) propose a framework for developing MKT. In their model, MKT develops when a teacher uses a key developmental understanding (Simon, 2006) of a mathematical idea to consider what students might understand about the idea, how they might come to a deeper understanding including types of learning activities to support that deepening of understanding, and the ways that new understanding positions students to learn other mathematical ideas. Implicit in this framework is the notion that teachers need to examine their own mathematics in relation to the logic of the discipline prior to considering students' mathematics. Thus, an understanding of students' mathematics follows teachers' development of their own mathematics. Further, the work of developing teachers' MKT starts with attention to SMK.

Similarly, Silver, Clark, Ghousseini, Charalambous, and Sealy (2007) outline a cycle of PLTs to develop teachers' MKT that use practice-based materials and always begins with an activity in which teachers solve a mathematics problem themselves. Next, teachers individually read and analyse a narrative case followed by a whole group discussion and concluding with collaborative work where teachers consider implications for their own practice. The authors note that the use of practice-based materials in this PLT cycle "integrates and interweaves several domains of knowledge germane to teaching: mathematics, pedagogy, and student thinking" (p. 266).

Contrary to this notion of first developing teachers' own understanding of mathematics, Phillip (2008) and Phillip, Thanheiser, and Clement (2002) suggest that in the case of elementary teachers, it is important to attend to the student prior to the mathematics. These authors propose that children are at the center of what elementary teachers care for, and therefore elementary teachers see the mathematics through the child. For Phillip and colleagues, the work of developing elementary teachers' MKT starts with a focus on students, which we interpret as a focus on the logic of the learner and, therefore, a focus on PCK.

In reporting findings from a mathematics professional development program with experienced teachers, Swan (2007) lists a set of general principles taken from a review of the literature for the design of tasks—whether they focus on PCK or SMK. These principles indicate that tasks need to include a focus on significant cognitive obstacles, understand and build from students' prior knowledge, and create "surprise, tension, and cognitive conflict" (p. 219). Similarly, Smith and Boston (2009) suggest that mathematics professional development that embraces a social constructivist perspective is built around PLTs that take into account teachers' prior knowledge and beliefs and purposefully create cognitive conflicts between teachers' prior views and new conceptions.

Learning Trajectory Professional Learning Tasks

In her review of a special issue of the *Journal of Mathematics Teacher Education* entitled "The Role and Nature of Mathematics-Related Tasks for Teacher Education", Zaslavsky (2007) compared the creation of mathematics PLTs to design experiment research. Initial selection or creation of a PLT is informed by professional literature, theories of learning, and personal experiences. Iteratively, PLTs are implemented and refined. Our current research project involves partnering with elementary grades teachers in a professional development setting designed to support their learning of the equipartitioning learning trajectory (Confrey, 2012) and therefore the development of their MKT around LTs. In the context of this work, we conduct our design experiment. The overall research question guiding our work focuses on understanding the ways in which teachers use their existing MKT to engage with the PLTs designed to support their learning of the LT. The purposeful use of a design experiment methodology within this professional development setting is meant to provide "systematic and warranted knowledge about learning and to produce theories to guide instructional decision making" (Confrey, 2006, p. 136). Design experiments "entail both 'engineering' particular forms of learning and systematically studying those forms of learning" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9). In the case of this study, we are "engineering" LT PLTs.

In creating our PLTs, we begin with a two-part learning conjecture to guide our design work: (a) PLTs that focus on the logic of the learner engage teachers in using MKT to learn about LTs; and (b) despite the tasks' focus on the learner, teachers use both pedagogical and subject matter knowledge domains of MKT to learn about LTs. The first part of our conjecture builds on the work of Phillip et al. (2002) and Philipp (2008) and stands in contrast to Silverman and Thompson's (2008). It indicates that our design of PLTs begin with attention to elementary teachers' PCK. The second part of our conjecture highlights our claim that teachers' MKT about LT spans both the PCK and SMK domains, combining the findings from Mojica (2010) and Wilson (2009). Further, it recognizes that elementary teachers' own content understanding is often underdeveloped, making it important for PLTs to provide teachers with opportunities to learn about SMK despite the initial focus on PCK.

Together, our conjectures indicate that PLTs focused on the logic of the learner engage teachers in using all domains of MKT as they learn about an LT. Guided by this conjecture, we propose and use in our own research work a set of design principles to develop our PLTs for LTs. They state that LT PLTs: (a) attend mostly to the PCK aspect of the LT, (b) embed opportunities for teachers to examine all aspects of their MKT, (c) employ instructional sequences that start with practice-based activities that challenge elementary teachers' views of students' mathematics and mathematics learning, and (d) use artifacts similar to the ones researchers used in developing the LT to highlight the logic of the learner.

The first two principles are informed by our theoretical perspective on teacher knowledge and follow from our learning conjectures. PLTs that are closely aligned with teachers' daily practices (Smith, 2001) frequently draw upon their PCK domains. Nonetheless, purposeful design may provide opportunities for teachers to engage both their PCK and SMK domains when learning about LTs. The third principle draws on literature on mathematics PLTs. LT PLTs should be grounded in teacher practice (Smith, 2001), create surprise or cognitive conflict (Swan, 2007), and allow for discussion and opportunities to consider learning in relation to their own practice (Silver et al., 2007). The final principle is based the work of researchers using various materials for professional learning including clinical interviews, video recordings, and analysis of student work.

An Example

In what follows, we provide a brief example of how we used our design principles from above to create LT PLTs as sequences of activities to engage and support teachers in developing their understandings of single or coupled levels of the LT. These PLTs and the ways in which teachers learn from engaging with them are the focus of our design research.

Each sequence begins with a challenge where we pose a question related to students' mathematics and present teachers with artifacts from practice (Smith, 2001) including videos of clinical interviews with children or student written work on diagnostic assessment tasks. The goal of these challenges is to problematize teachers' current views of the logic of the learner, focusing on teachers' PCK. Following the challenge, teachers engage in an exploration activity that requires them to consider all aspects of their MKT to examine and resolve the challenge at hand. Resolutions from these explorations are formalized in whole-group discussion and are used in an application closely related to instruction, such as work with curriculum materials or examination of videos from whole-class instruction.

One of our LT PLT sequences aims for teachers to learn three cognitive processes children must coordinate when equipartitioning as described in two levels of the LT. These three equipartioning criteria (Confrey, 2012) include creating the correct number of groups or parts, creating equal-sized groups or parts, and exhausting the original collection or whole. We begin the sequence by challenging teachers with the question, "Based on their written work, what do these students know about equipartitioning?" and providing them with carefully selected written responses to diagnostic assessment items which exhibit different partial understandings of the three criteria. In whole group discussion, teachers explore the similarities and differences among the work samples. Guided by the facilitator, the teachers formalize their observations as the three equipartitioning criteria and then apply this learning to an analysis of a new set of written work samples.

The goal of this LT PLT is to problematize teachers' current PCK about equipartitioning (principle a) by challenging teachers' views of students' mathematics and learning in the practice-based activity of analysing student written work (principle c). It engages teachers in analysing responses to diagnostic assessment items used by researchers when developing the LT (principle d). Finally, it provided an opportunity for teachers to engage with their SMK as well as their PCK, specifically the part-whole relationship that exists only when the three criteria are met (principle b).

Conclusion

Research on LTs is quickly moving from an agenda for examining student learning to an agenda for promoting teacher learning and researchers need to attend to the ways in which teachers come to learn about the recently developed, empirically-tested descriptions of how students learn about specific mathematics content topics over time. The focus on teacher learning highlights the need to attend to teachers MKT in relation to LTs. In our design experiment research, we claim that teacher knowledge of LTs expand all domains of MKT, and we have proposed a set of learning conjectures and design principles to guide our work in developing and empirically testing a set of LT PLTs. We contend that our learning conjectures and design principles are an initial attempt at explicitly articulating required features of PLTs that aim at supporting teacher learning of LTs.

Acknowledgements

The research reported is based upon work supported by the National Science Foundation under grant number DRL-1008364. Any opinions, findings, and conclusions or recommendations expressed in this report are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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This document was originally published as Wilson, P. H., Sztajn, P., & Edgington, C. (2012). Designing professional learning tasks for mathematics learning trajectories. In T.-Y. Tso (Ed.), *Proceedings of the 36th annual meeting of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 227-233). Taipei, Taiwan: PME.

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Received: September 2012. Accepted: January 2013.
Handle: http://hdl.handle.net/10481/24791