CONSTRUCTION AND MODIFICATION OF THE AUTONOMY OF SCHOOL MATHEMATICAL KNOWLEDGE IN PORTUGAL

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During the second half of the nineteenth century and early twentieth century the discipline of secondary mathematics was gradually built in Portugal and certified teachers, textbooks, programs, special teaching techniques emerge. This consolidation process ends with the emergency of school subjects that develop some kind of autonomy as Chervel puts it. This article discusses how the school discipline of mathematics in secondary education in Portugal was set. This process is inseparable from teacher training and so we will observe professional legitimation processes paying special attention to ways in which autonomy has been building and modifying over time.

Keywords: Autonomy; Cultural history; Mathematics education; School subjects

Construcción y modificación de la autonomía del conocimiento matemático escolar en Portugal

Durante la segunda mitad del siglo XIX y principios del siglo XX la disciplina de las matemáticas en secundaria fue construida gradualmente en Portugal y emergen libros de texto, programas, técnicas de enseñanza especiales y la certificación de maestros. Este proceso de consolidación termina con el surgimiento de disciplinas escolares desarrolladas con algún tipo de autonomía como la planteada por Chervel. Este artículo describe cómo se creó la disciplina escolar de las matemáticas en la educación secundaria en Portugal. Este proceso va unido a la formación docente y, por tanto, se observan procesos de legitimación profesional, prestando especial atención a las formas en que la autonomía se ha estado construyendo y modificando en el tiempo.

Términos clave: Asignaturas escolares; Autonomía; Educación Matemática; Historia cultural

In a fundamental text written in 1988, André Chervel argues about the legitimacy of historians to study school subjects *per se*. According to him, for mainstream History

of Education, school subjects were an unproblematic application to the school context of knowledge developed elsewhere and teachers a professional group constrained either by the great options of educational policy—themselves the result of large movements socio-economic—or by knowledge (science and culture) externally generated.

Drawing on studies on the development of grammar in France, among others, Chervel will show that school subjects have the ability to generate their own knowledge that escapes those determinants and that, at least in the case of grammar, even generate scientific knowledge itself. Historians can only gain understanding if they study school subjects on their own, that is, endowed with a large degree of autonomy from external constraints to school.

Almost simultaneously, Lee Shulman, also in a founding text of 1986, faced with the need to evaluate the job performance of teachers, argues that teaching knowledge has special characteristics which distinguish it (and, in a way, give autonomy) either from general pedagogical knowledge, or from scientific knowledge, and named it pedagogical content knowledge. To him this knowledge, deeply attached to professional practice, is very different from the other two either in content or its nature. In a further text (1987) Shulman characterized as "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987, p. 8).

These two researchers, while working in different areas, highlight the specific nature of the knowledge associated with particular school disciplines. In this article I will term it school knowledge.

This study focuses on the gradual changes of autonomy of school knowledge. Academically, it walks on the borders between the professional development field, particularly teachers, and the historical study of school disciplines. Looking beyond the mere compilation of intentions expressed in legislation pieces, I chose a methodology akin to the cultural history of Peter Burke valuing the symbolic and its interpretation (2005). As states Chartier,

> *This story should be understood as the study of the processes with which a sense is built. Departing from the old idea that endowed texts and literary pieces with an intrinsic, absolute, and unique sense—which criticism was required to identify—it addresses practices that plurally, contradictorily, give meaning to the world.* (Chartier, 1990, p. 27)

Chartier pays particular attention to the concepts of representation and practice. Representation includes the classification processes that produce the categories through which reality is (contradictorily) constructed by different groups. Practices of these groups give them a certain social identity that in turn will condition the representations themselves, and integrates institutionalized and objectified forms that mark the existence of this group (1990, p. 23). Dominique Julia (1995) also calls attention to these two categories.

A central problem of cultural history is the relationship between the representations and practices. Chartier puts forward the concept of *appropriation* designating the distinct ways in which social groups incorporate cultural elements. "Appropriation (...) aims at a social history of interpretations, sent to its fundamental determinations (which are social, institutional, cultural) and inscribed in the specific practices that produce them" (Chartier, 1990, p. 26). These practices "produce order, statement, distances, divisions" (pp. 27-28). Similarly, representations "have their own energy convincing that the world or the past is, indeed, what they say they are. In this sense, they produce gaps fracturing societies and incorporate them in individuals" (Chartier, 2007, p. 73).

In summary, this work accompanies the preparation and progressive autonomy of school knowledge by studying the way teachers, i.e., the experts, holders of such professional knowledge, as Shulman would say, will constitute "their" school discipline, as would Chervel pose it, and become legitimate holders of this knowledge. Particular attention will be given to the construction of meaning and a (feasible) analysis of practices. Following other work, I will study the case of the constitution of the mathematical discipline in Portuguese secondary school.

Studying the configuration of this knowledge will miss the contribution of two sets of professionals: teachers of private schools and individuals who taught in private that had an important expression until 1895. I will not include the emergence of the academic field that discusses the problems of teaching and learning mathematics, that some term mathematics educators (Furinghetti, Matos, & Menghini, 2013) and will take place essentially in the second half of the twentieth century.

THE AXES OF ANALYSIS

The professionalization of Portuguese teachers was studied by Nóvoa (1987a). Organizing it around the central axis social and economic status of teachers, and in a formulation but precedes Chartier or Julia, he distinguishes two dimensions.

> *Building a body of knowledge and techniques proper and specific to the teaching profession, that is in continuous reformulation.*

> *The organization (explicit or implicit) of a set of standards and values that should guide the exercise of the teaching profession and even the everyday activity of teachers.* (Nóvoa, 1987b, p. 416, italics in original)

His work, especially focused on primary education, leads him to distinguish four steps in this process of professionalization.

Full-time commitment (or as the *main occupation*) *to the teaching activity*, making teaching the way of life of an increasingly specialized professional group, the teachers.

Establishment of a legal framework for the teaching activity, through the legal imposition for teachers to possess a licence (and/or a degree) given by the State in order to teach.

Creation of specific institutions for teachers training whose mission is to pass on to future teachers the knowledge and techniques, norms and values, akin to the teaching profession, through a "role-transition" (passing from the role of a student to the role of a teacher) rather than "role-reversal" like most other professions.

Establishment of professional teacher associations, usually union-like organizations that play a key role in the development of an esprit de corps and the suport of the social and professional status of teachers (Nóvoa, 1987b, p. 417, italics in original).

The centre of my work is in school knowledge and the discussion of its autonomy. Accordingly, I will take into account Nóvoa's categories but I will not establish close temporal boudaries. Given that the study is focused on the discipline of mathematics, we need to discuss first the constitution of mathematics school knowledge in secondary education and the development of didactical methodologies for their teaching. Secondly, I will examine the constitution processes of professional mathematics teachers for secondary education and its impact on the development of this knowledge. Finally I will review the limits of the autonomy of these professionals, again focused on the consequences for the preparation of school knowledge.

THE DIFFERENTIATION OF THE TOPICS

The political regime that emerges after the civil war of the 1830s and the consolidation of a constitutional monarchy will gradually differentiate a nationwide public secondary education system (Nóvoa, Barroso, & Ó, 2003) and will be within these new schools essentially the *Liceus* created from 1836, which will be incorporated new school subjects, including mathematics in secondary education (Matos, 2013, 2014).

The first curriculum of *Liceus*[1] includes the discipline arithmetic, algebra, geometry, trigonometry, and drawing and the curricular organization was determined by each school board (Aires & Santiago, 2014). In the first decades, the distinction between *Liceus* and higher educational institutions was unclear. The mathematics courses, in particular, were both taught at the University (Coimbra) or the Polytechnics Institutes (Lisbon and Oporto) and shared the same teachers and textbooks, usually the works of Bézout (Matos, 2014).

When we look at the evolution of the names of mathematical disciplines in *Liceus* (Table 1) we note that, unlike other disciplines, only after 1895 the name was fixed in "Mathematics". Although the legislation always refers to *one* discipline, the chosen names join independent themes that sometimes even have distinct textbooks. The unity of mathematics will only be claimed by the Modern Mathematics movement in the second half of the twentieth century (Matos, 2013).

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[1] The term *ensino secundário* (secondary school) is used in the legislation and includes public classes, municipal secondary schools and private schools. *Liceus* are public secondary schools.

Table 1

Name of the Disciplines Related to Mathematics in the Legislation Concerning the Portuguese Secondary School

Date	Name
1836	Arithmetic, algebra, geometry, trigonometry, and drawing
1844	Arithmetic and geometry with applications to the arts, and first algebra concepts Geometry and mechanics applied to the arts and crafts
1854	Arithmetic, elementary algebra, elementary synthetic geometry plane Trigonometry principles and mathematical geography
1860	Arithmetic, the four operations on integer and fractional Arithmetic, plane geometry concepts and their usual applications Elementary mathematics
1868	Mathematics
1870	Elementary mathematics
1872	Mental calculation and four operations (1st part of mathematics) Arithmetic practice (1st part of mathematics) Mathematics
1880	Arithmetic, plane geometry, algebra principles and bookkeeping Algebra, geometry and space trigonometry
1883	Arithmetic, algebra, geometry, trigonometry and principles of bookkeeping and accounting
1886	Elementary mathematics
1895	Mathematics Arithmetic, elementary algebra and plane geometry Algebra, geometry in space, trigonometry, elementary and cosmography

Even these designations were not stable. Occasionally independent classes[2] were established with different names. For example, in 1861 is created in Elvas, who did not have a *Liceu*, a class of "arithmetic and geometry with application to industry, read in two-year course with the rational and moral philosophy and principles of natural law" (*Coletânea Oficial da Legislação Portuguesa*, 1861, p. 430). In the late 1850s the first mathematics books explicitly aimed at *Liceus* appeared:

♦ *Elementos de Aritmética* (Elements of Arithmetic) by Rufino Guerra Osório (1810-1879) that publishes in 1854 its 2nd edition.

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[2] The Portuguese term is "cadeira", literally chair or "aula", class.

♦ *Elementos de Trigonometria Rectilínea, e sua aplicação à topografia* (Elements of Rectilinear Trigonometry, and its Application to Topography) of José Joaquim Manso Preto (1823-?), published in 1856.

♦ The 2nd edition of *Geometria elementar teórica e prática* (Theoretical and Practical Elementary Geometry) of Francisco de Castro Freire (1809-1884) and Rodrigo Ribeiro de Sousa Pinto (1808-1893) from 1859; and others will follow.

All these manuals were approved for *Liceus*, some by the *Conselho Superior de Instrução Pública* (Superior Council of Public Instruction) that supervised primary and secondary schooling. All these authors were professors at the Faculty of Mathematics of the University of Coimbra and it is likely that some of the texts were also used at the University.

In 1865 is published the first book of math exercises, *Problemas de Álgebra para Exercícios dos Princípios Gerais desta Ciência* (Algebra Problems to Practice the General Principles of this Science), written by Marcal António Carvalho who was a teacher at *Ateneu Lisbonense das Ciências e das Letras*, a private school in Lisbon that included primary and secondary schooling.

Since 1870, when the first programs for secondary education were published, a legal corpus that framed mathematical knowledge for secondary education existed (Aires & Santiago, 2014) and of course the next decades will bring changes in their content.

From 1894 a reform directed by the Minister Jaime Moniz homogenizes the structure of the Portuguese secondary school establishing it definitely as an intermediate degree between the primary school and the university (Nóvoa et al., 2003). Mathematics programs of this reform were published in 1895 and contained a detailed list of contents that are structured around arithmetic, geometry, algebra, and cosmography. In the first three years the program expanded arithmetic from primary school and included powers, multiples, dividers, remainders, fractions and decimals, square roots, proportions and the rule of three. Geometric figures were also considered, including properties of straight lines, circles and triangles.

Algebra began in the fourth year with a discussion of formulas, monomials, polynomials and its operations, including division. First-degree equations, systems of equations, inequalities and integer solutions of first-degree equations were also mentioned. Geometry included a detailed study of the properties of polygons and constructions, as well as circles. In the fifth year, arithmetic and geometric progressions and their sums were studied, including a brief discussion of limits. Logarithms were applied to the computation of interests. The geometry was focused on ellipses, parabolas and hyperboles. The Cartesian coordinates were applied to the study of the equation of the straight line. In the sixth year systems of equations and second degree equations were discussed, imaginary numbers and the geometry of space was studied. The last year included continuing fractions, trigonometry and cosmography (Aires and Santiago, 2014).

Jaime Moniz's concern with homogenization introduces unique books for secondary schools, a decision that was removed in 1905. The winner of the successive contests is Joaquim de Azevedo Albuquerque (1839-1912), a mason under the name of Condorcet, that while professor at the Polytechnic Academy of Oporto had been a teacher of the *Liceu* of Oporto for 14 years.

Of course, over time there were variations of the topics I listed and although large areas of arithmetic, algebra, geometry and trigonometry are present since 1870 there have been significant fluctuations. Before and after 1895 programs include and exclude topics such as geometric drawing, accounting, topography, mathematical geography or cosmography (which oscillated between the disciplines of mathematics and history), subjects that for many years are not included in Portuguese mathematics programs.

These changes are due to adjustments in boundaries among school disciplines. But there are also variations due to fluctuations in mathematical content. Some topics appear as rational arithmetic in 1872; or analytic geometry in 1886—the latter giving rise to disagreements about its purpose (should it be an extension of geometry or a preparation for the study of analysis?) and sometimes it is withdrawn from the programmes; or the study of the analysis that started in 1905. A brief introduction of the study of integrals was included in 1918, but removed in 1926. The study of derivatives and analytic geometry was eliminated between 1936 and 1948, following the choice of the authoritarian regime in lowering the quality of education at all levels (Aires & Santiago, 2014). Only after World War II these two topics were again studied in *Liceus* (Almeida, 2013). Major changes to this curriculum will take place during the second half of the twentieth century following the trend of the Modern Mathematics movement (Matos, 2013).

In conclusion we can consider that from 1870 on a national programme for secondary mathematics is established. Though with some variations due either to the adjustment of boundaries among school subjects, or additions and deletions of mathematical topics, it became possible to clearly identify a body of knowledge that separates this discipline from mathematics the university level.

This distinction is not only a matter of a list of topics. The construction of this mathematics curriculum for secondary education also involved the creation of didactic sequences, exercises, representations, developping interconnections among mathematical topics and associations with other school topics or matters external to schools, many of them expressed in textbooks. These elements exist in a school setting and only make sense in that context, as Chervel states (1988).

Even in the case of Portuguese textbooks of this time, far from a prior norm (legislative or academic) that is appropriated by the authors of textbooks produced for a given practice, what happens is rather a hybrid process in which the books precede the programme itself and where, especially from 1860, the intervention of the Conselho Superior de Instrução Pública (which included university professors which were also textbook authors) will gradually condition the books themselves and moves them apart from higher mathematics.

THE METHODOLOGICAL RECOMMENDATIONS

In addition to the development of adequate content, the creation of a school subject also covers the development of methods of teaching, which refers to the teaching practices. We do not have elements that allow us to discuss these practices in the Portuguese secondary education of the nineteenth century. We can, however, search for recommendations in the legislation that recommend teaching methods—the *norms* of Chartier and Julia. Thus, we observe that the first published recommendations are still influenced by the Jesuit tradition. For example, the Regulation for Secondary Education of 1869 (*Regulamento para o Ensino Secundário, Diário do Governo*, 133, 1860) states:

Article 30. From the two hours that the class lasts, teachers will employ at least one to hear as many students over the previously lesson, and the rest of the time to give explanations as they deem appropriate for the full understanding of the doctrines which were the object of the lesson given in that day or the one that students have to study for the next day of school.

Article 31. In all classes there will be exercises or written themes, which will be discussed and amended by the teacher, out loud, and to the whole class.

Article 33. There will be a monthly exam, oral or in writing in all classes.

These recommendations are repeated in subsequent legislation with minor changes. Only in 1880 programs start to include notes with more detailed didactic recommendations specific for mathematics. For example the program of 1880 suggested for the first year of *Liceus*.

> *Arithmetic teaching in this year is entirely practical, and the study of the rules for arithmetic calculation must be accompanied only by the most simple theoretical notions absolutely necessary for the intelligence of these rules to the exclusion of any synthetic and abstract definitions that the circumstances in which it must be assumed the student would be simple memory exercises. The geometry of education includes demonstrations but only the simplest, whenever possible by superposition and based on evidence.* (Decreto, Diário do Governo, 240, 1880, p. 2747)

Previous recommendations focused on the temporal organization of classes and contents, highlighting the repetitions of topics and the evaluation. Now the concern is about student understanding, memorization is devalued (in the case of arithmetic) and evidence as the root of conviction is valued (in the case of geometry). The New School movement also begins because be felt in secondary education (Matos, 2013).

The 1894 reform deepens this trend. Mathematics is seen as a particularly valuable discipline to introduce students to a specific kind of thinking and emphasize the importance of simplicity, clarity, accuracy, and the possibility of complex intellectual constructs. The mathematics course of this reform consolidates the modern mathematical content essential for life and to prepare for the study of science in university

courses. As for teaching methods, the programs include "Notes" which recommended the use of intuitive and practical approaches in the early years.

Belying the current common sense that stigmatizes the "traditional teaching" almost all subsequent programs highlight the importance of the disciplined use of rigorous mathematical methods, emphasize the importance of intuition and practice and while valuing mental and written calculations warn against the overemphasis on memorization.

There are, however, two deviations from this trend. Between 1930 and 1947, and agreeing with the ideological choices of the dictatorship that persecuted the prominent proponents of the New School movement, the programs, especially the first five years of *Liceus*, underscore almost exclusively the importance of repetition in exercises and omit references to intuition and the concrete (Matos, 2013). The second deviation occurs in the present moment in which official recommendations narrow school mathematics, by valuing student performance in targets evaluated in exams, and neglecting all the heritage of the New School pejoratively referred to as "romantic".

THE CONSTITUTION OF PROFESSIONALS

After addressing the creation of content and the development of methods of teaching secondary school mathematics I will now discuss the genesis of its professionals.

By the twentieth century there were no teacher training system for secondary education in Portugal. Since the mid-eighteenth century that access to public places was done through exams but if we look at the legislation produced throughout the nineteenth century, we find that secondary school teachers did not even had to have a university degree. Legislation itself puts the emphasis on obtaining the stability of an official position and not in candidates' scientific knowledge. Lourdes Silva (2002, p. 126) confirms, in the documentation she analysed most teachers do not state any prefixes to their name nor mention any academic title.

We know the procedures for getting access to these public places. In 1851, for example, legislation detailing the actions to be followed is published (*Coletânea Oficial da Legislação Portuguesa*, 1851, pp. 2-8). Candidates were examined in the field of the disciplines they applied, and on the methods of teaching, including exercises. Exams also included the simulation of a lesson. The subsequent legislation maintains this structure.

Who then were the teachers who taught mathematics at this time in *Liceus*? I will just give two examples. The first is José Adelino Serrasqueiro (1835-n.d.). Obtaining a bachelor's degree in Medicine and Philosophy in the University of Coimbra, he majored with honours and received several awards. Professor of Mathematics in the Central *Liceu* of Coimbra, he was an author of successful textbooks (Valente, 2002). The second is General José Nicolau Raposo Botelho (1850-1914), textbook author of mathematics textbooks and professor at the Army School, in the Central *Liceu* of Oporto, at the Normal School of Oporto, and director of the Royal Military College. In addition to having held several positions in the army, he was also Minister of War.

These are just a few examples of personalities who have excelled and who also taught mathematics at the secondary level, but it certainly does not represent the majority. Information on "common" teachers is scarce, but considering the rules for accessing the profession, we could assume that most would just have a secondary education and very few a university degree, particularly in the "scientific" areas, as in "humanities" a significant number of teachers had a clergy origin (Silva, 2002).

The assumption that the teaching profession requires specific training is present for primary schools since the early nineteenth century, but only in the twentieth century standards for accessing the teaching profession in the secondary education will change. In 1901 it is created a Qualification Course for Secondary Teaching (*Curso de Habilitação para o Magistério Secundário*) with duration of four years (Pintassilgo, Mogarro, & Henriques, 2010). The chosen model separated the basic scientific formation from training for the teaching profession. In the first three years prospective teachers would carry out scientific education at the University of Coimbra or both polytechnics institutes at Lisbon and Oporto. In the fourth year teacher training was held at the Higher Course on Humanities (*Curso Superior de Letras*) in Lisbon.

This model is improved with the creation in 1911 of Higher Normal Schools (Escolas Normais Superiores) attached to the Coimbra and Lisbon Faculties of Arts, which start operating in 1915. After a bachelor's degree in the areas of specialty, students attended a course two years similar to the Higher Course but now including an introduction to teaching practice in *Liceus*.

The Higher Normal Schools will play a central role in the training of professionals for the teaching of mathematics for the secondary schools. The course provided reflections either on the school mathematics or on methods for teaching the discipline. The training of primary teachers already included this aspect some years ago (Candeias & Matos, in press), but it's the first time it will be implemented at the secondary level. These reflections occur at the annual course of the first year General Methodology of Science Mathematics under the responsibility of university professors.

The second year was focused on initiation into the pedagogical practice that took place in a *Liceu* and had an important role for teachers. It consisted of two parts, the Special Methodology course taught by teachers from the *Liceu* and the teaching practice itself directed by the same teachers.

> *Since the beginning of the school year until December 24, applicants attend classes of the teachers of Liceus (...) where they are practicing, and whose teachers will give them the essential notions about the special methodology of the respective disciplines (...). The rest of the school year, teaching will be exclusively performed by the candidates, under the supervision of tutor teachers [professors dirigentes] who examine their corrections in writing exercises done by students, and always attend to their lessons, enlightening them with their advice* (Decreto com força de lei, Diário do Governo, 1911, 129, pp. 2081-2083).

The structure of these courses included collaborative spaces between university professors and teachers of secondary education, and gave the latter a key role in the formation of professional knowledge. It was a time when Pedagogy and Psychology were being established as experimental sciences and the course also included exercises of experimental pedagogy and child psychology studies, performed in psychology laboratories associated with Normal Schools (Pintassilgo, Mogarro, & Henriques, 2010).

Ismael Eduardo dos Santos Andrea (1879-1937) was the professor of General Methodology of Mathematical Sciences and illustrates the permeability between secondary education and the universities. He taught at several *Liceus* in the country, was a Professor at the Polytechnic School of Lisbon and Director of the Normal School of Lisbon. Politically involved, Andrea also left its mark in the teaching of mathematics problems and published several textbooks for secondary education. One of his "tutor teachers" was Domitila Hormizinda Miranda de Carvalho (1871-1966), then a professor at the *Liceu* Maria Pia in Lisbon, physician, teacher, writer and deputy. She was the first woman admitted to the University of Coimbra and completed medical school and the mathematics course.

The course at the Normal Schools ended with a State Exam (*Exame de Estado*) whose jury included the secondary school teachers who had accompanied the candidates. Among the assessment criteria was a "dissertation (...) on a didactic point of secondary education" (*Decreto com força de lei*, *Diário do Governo*, 1911, 129). We know the titles of about 40 dissertations related to the teaching of secondary mathematics. Most of these titles can be found in Gomes (1989). The study of these dissertations lets us know how the legislator's intentions were appropriated.

Let us take two examples. Francisco Ferreira Neves (1892-1984) was for many years a professor at Liceu of Aveiro. Successful textbook author, intervening in local politics, held his training at the Normal School of Coimbra which he ended in 1918. His dissertation Mathematics in secondary education. Didactic considerations (Neves, 1918) is actually a book of 100 pages. It discusses the nature of mathematics, proving to be an adept of Poincaré's ideas. Then he proceeds with an extended comparison among teaching methods: the *heuristic* linking the Pestalozzi and Socratic dialogues; the *didatic*, which combines the exposure of the finished science; and the *laboratory*, discussed at length based on John Perry's and E. H. Moore's work. He ends with the presentation of methods to teach specific mathematical topics, quoting Poincaré several times.

Let us take a second example. João da Conceição Dâmaso Rego (1891-?) was a professor at *Liceu* Camões in Lisbon and attended Normal School in Lisbon. His dissertation *The differential and integral calculus in liceus* (Rego, 1918) deals with an innovative theme (analysis), proposed in fresh indications from the ICMI and partially integrated in the Portuguese programs since 1905. He refers Jules Tannery, a proponent of the introduction of the analysis in the French curriculum, in the first paragraph. The development of the theme is supported by numerous quotes from a report submitted by Emanuel Beke to the 1914 ICMI in Paris (*Les resultats obtenus dans*

l'introduction du calcul différentiel intégrale et dans les classes superieurs des établissements secondaires) four years ago.

The dissertations of these schools, as well as the ensuing dissertations from 1930 produced from a different training model that I will not analyse here, constitute the first systematic body of knowledge on methods to teach mathematics in secondary education published in Portugal. Covering all the themes of the program, these studies with dozens of pages show us how the relationship between legal rules (the norms) and the training practices was woven, that is, they show us how teachers and students have appropriated the rules and materialized them in practices. The texts include didactic explorations of innovative themes, namely the teaching of analysis or the use of materials. It is worth to note that they were produced in the context of training and therefore subject to exchange of ideas among professionals who certainly helped to deepen the ideas of tutor teachers, prospective teachers, and university professors and therefore to refine the pedagogical content knowledge, as Shulman would put it.

Autonomy and School Knowledge

The intent of this text is to understand how the autonomy of secondary school mathematics discipline was established. I argued that it was born as a specific body of knowledge and the simultaneous advent of professionals, endowed with the legitimacy of a specialized training. These professionals, confronted with their teaching practice, will refine this knowledge. This characterization allows us to support the vision of the autonomy of school disciplines expressed by Chervel, that is, the mode of production of this knowledge assures us that it is specific to each discipline, and in this sense autonomous. Shulman argues in similar vein, based on the epistemological nature of pedagogical content knowledge.

However, in what sense can we say that teachers are autonomous in the production of school knowledge? This is a different question but that intersects with the previous one. Even without entering the debate about the characteristics of the teaching profession, namely whether or not it is indeed a profession or discuss their autonomy in general[3], we can deepen our understanding of the autonomy of teachers in the production of school mathematics at least in what concerns the Portuguese case.

We know something about the ways in which the socialization of Portuguese teachers occurred, in particular, the forms of professional association that allows for the dissemination of a sense of belonging to a community. In the transition decades of the nineteenth to the twentieth century, primary teachers were the main pursuers of teaching associations, which led to the creation of the first organizations of the class (Nóvoa, 1987b). However, in secondary education

> *The late associative organization of secondary school teachers, as they constitute a rather small group and the fact the taught an elite and having a rela-*

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[3] A discussion about this themes can be found in Pintassilgo (2008).

> *tive social prestige are some of the specifics of this sector of teachers, which cause them to develop a kind of associations distant from unionism and also only partially mutual.* (Pintassilgo, 2008, p. 93)

Joaquim Pintassilgo also points that the very sharp division between teachers of *Liceus* and technical schools, the care effective teachers put in setting themselves apart from lower categories of teachers lead to a difficulty to distinguish a single professional identity of secondary school teachers (2008).

However, between 1927 and 1931 five meetings of secondary education were held, preceded by intense associative activity and the foundation of the prestigious magazine Labor, issued from the teachers' initiative of the *Liceu* of Aveiro. The political context will become, however, increasingly unfavourable, and trade unions will be either prohibited or integrated in the corporative fascist regime under the control of the government. What could have been a professional association of secondary school teachers never saw the light of day. Labor, however, was published during the entire Salazar period (it ceased publishing in 1973), witnessing the reflections of secondary education teachers in building a teaching profession.

The magazine contains well over a hundred articles related to mathematics teaching. In addition to the mathematical deepening of school subjects, suggested teaching methods, and proposals for the use of innovative materials, we find a very lively debate on the new secondary education programs of 1948 as well as some articles reflecting on modern mathematics (Almeida, 2013).

Other newspapers also contain reflections on the teaching of mathematics, including Gazeta de Matemática, a journal created by mathematicians and Vértice. There were also "official" magazines which contain important items.

IN CONCLUSION

This article sought to draw a picture of the development of mathematical school knowledge for secondary education with particular emphasis on its autonomy, as is suggested by Chervel and Julia. I believe I have argued that this autonomy has been developing in the first place through the establishment of programs, i.e., the choice of topics and their presentation sequence that differentiate them either from those of the university or from primary school. The appearance of textbooks even those written by members of the academy, precedes and gives consistency to this knowledge. Secondly, and simultaneously, teaching methodologies (representations, motivations, teaching sequences, exercises, support procedures for pupils understanding, etc.) were being developed of which only have glimpses in legislation. Thirdly, a key step is taken by establishing teacher training procedures, especially because of the way these procedures required trainers and trainees, to reflect on (and construct) this knowledge. A final step occurs when we find such reflections now in the context of opinion pieces published in professional journals.

Although it could be argued that the autonomy of teachers was very limited because of its functional dependence of the Government, especially during the Salazar period, data, particularly as it concerns school mathematical knowledge, seem to contradict this view. Although beyond the scope of this text, we find public debates, sometimes very intense, on educational topics (e.g., exams). The regime's pressure did not prevent for example that remarkable reflections on the teaching profession had expression in Labor precisely starred by teachers of mathematics (Pintassilgo, 2002a, 2002b). It would be also interesting to compare the degree of autonomy enjoyed by these teachers as they developed school knowledge with today's autonomy allowed in Portuguese schools, dominated by the focus of exam results and competition among professionals.

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