ROUTE DESCRIPTION IN IWAIDJA: GRAMMAR AND CONCEPTUALISATION OF MOTION

Cris Edmonds-Wathen

This study focussed on the effect of grammar of Iwaidja, an indigenous Australian language, on mathematical conceptualisation. It investigated route description in Iwaidja. Spatial concepts such as direction, height and movement in relation to another object are briefly described using examples. Differences between English and Iwaidja are used to illustrate the some of the impact of grammar on mathematical conceptualisation. The implications are discussed in terms of how understanding these grammatical features can help teachers, especially when children are not fluent in the language of instruction, as well as providing keys to cross-linguistic investigations of mathematical cognition.

Keywords: Cognition; Culture; Grammatical structures; Indigenous; Mathematical conceptualisation; Spatial language

Descripción de ruta en iwaidja: gramática y conceptualización del movimiento

Este estudio se centró en el efecto de la gramática de iwaidja, una lengua indígena de Australia, en la conceptualización matemática. Se investigó la descripción de ruta en esta lengua. Conceptos espaciales tales como dirección, altura y el movimiento en relación con otro objeto, se describieron utilizando ejemplos. Diferencias entre el inglés y el iwaidja son usadas para ilustrar el impacto de la gramática en la conceptualización matemática. Las implicaciones consideran cómo la comprensión de estas características gramaticales puede ayudar a los maestros, sobre todo cuando los niños no hablan con fluidez el idioma de la instrucción, así como proporcionar claves para investigaciones translingüística de la cognición matemática.

Términos clave: Cognición; Conceptualización matemática; Cultura; Estructuras gramaticales; Indígenas; Lenguaje espacial

Many children learn mathematics in languages in which they are not yet fluent. In Australia, most indigenous language speaking students are taught by English-speaking teachers in English. The ways these children's languages conceptualise the world may be different to what is assumed in their mathematics education. Improving mathematical learning outcomes for these students requires non-indigenous teachers to better understand how their students think and talk mathematically. For teachers to better understand the students' mathematical concepts, we can examine the languages that shape those concepts. Grammatical structures provide clues to how speakers of different languages understand events. Mathematical language has specialised words and grammatical structures but it is built out of everyday language. The linguistic structures of individual languages affect how languages can be used and developed for mathematical thinking. Linguistic structures of different language are an influence on the cognitive development of children who speak those languages. Describing the variety of mathematical expression may also assist other teachers who are teaching in multilingual classrooms, as well as enrich researchers' appreciation of the scope of variations in mathematical conceptualisation in different languages.

This paper considers how the expression of route descriptions in Iwaidja, an indigenous Australian language might affect mathematical conceptualisation. Part of everyday language, spatial language such as route description impacts directly on overtly spatial mathematical domains, such early years descriptions of position and movement, mapping and coordinate systems. Spatial language is also used as a basis for the expression of other domains; hence differences in spatial language can affect expression in mathematics domains that are not directly spatial.

LANGUAGE OF INSTRUCTION IN MATHEMATICS

In some places where the language of mathematics instruction is not the first language of the students, the teacher does not share the students' first languages, particularly students who are immigrants or the children of immigrants (Staats, 2009) and indigenous students in (post-) colonized countries (Edmonds-Wathen, 2015; Lunney Borden, 2011, 2012). Instructional strategies are needed for when the teacher does not speak the students' languages (Phakeng & Moschkovich, 2013). One approach is to explore mathematical concepts and expressions in students' languages, such as Staats (2009) with Somali students in the United States and Lunney Borden (2012) with Mi'kmaw students in Canada. Staats (2009) explored the cultural meanings of the mathematics terminology of her undergraduate Somali mathematics students. She describes how the metaphorical origin of Somali mathematical terminology collapses a distinction between formal and informal mathematical words, stimulating mathematical thought by

the association of mathematical concepts with their metaphorical referents. Lunney Borden (2012) argues that understanding how mathematical concepts are talked about in the Mi'kmaw language can help teachers meet the mathematical needs of their Mi'kmaw students.

GRAMMAR OF MATHEMATICAL LANGUAGE

The language of mathematics is more than vocabulary, and includes grammar as well as many other features of how language is used. Students need to learn the *mathematics register*, "a set of meanings that is appropriate to a particular function of language, together with the words and structures that express these meanings" (Halliday, 1978, p. 195). One typical feature of the mathematics registerin English is the use of dense noun phrases, where processes are nominalized. The verbs, rather than representing process, represent relations between processes. Mathematical language in English tends to be somewhat static, directed towards describing something that exists in a timeless moment.

There has been little cross-linguistic investigation of grammatical features of the mathematics register in languages different to English. Different languages use their word classes such as nouns and verbs in different ways, which may affect how mathematical concepts and processes are handled. For example, Barton (2009) describes how in different languages numbers belong to different grammatical categories and there are different ways of locating oneself in space. Barton focuses on examples from languages that have not been part of the development of what he calls NUC-mathematics (the Near Universal, Conventional mathematics), claiming that some languages are more consonant with mathematics than others as a result of co-evolution of these languages and mathematics. Barton also concludes that each language hosts its own mathematical world, challenging his readers to think about the mathematical possibilities of different languages.

One of Barton's (2009) examples was inspired by Pinxten, van Dooren and Harvey's (1983) linguistic-anthropological description of Navajo spatial language. Navajo can be described as a *verb-based* language, compared to a *noun-based* language such as English. In the Navajo language, shapes are verbs. There is no circle, there is circling. The Navajo world view is premised on seeing the world as dynamic, be always changing and moving, and is therefore less suited to being divided up into parts than the static Western world. Pinxten et al. describe a proposed teaching program responding to the Navajo worldview, emphasising that these cultural practices should be kept culturally situated even as mathematical abstractions were developed from their construction. Barton (2009), working with John Mason, explored mathematical implications of treating shapes as verbs, as actions, calling it Action Geometry. For example, the static view of a circle is all the points that are equidistant from a centre point,

forming a planar shape. In a dynamic view a circle is movement with a constant speed and with a constant rate or turn: "Circling is actually a special case of spiralling" (p. 31). Barton stresses that Action Geometry is not an actual practice of the Navajo and does not represent a culturally situated response to the Navajo worldview, but was invented by mathematicians, inspired by Pinxten et al.

There are many other languages in the world which could be described as verb-based, such as the Mi'kmaw language. Battiste and Henderson (2000) proposed that teachers of Mi'kmaw students take this into account.

> *Children who are raised with the Mi'kmaw language are used to focussing on verbs and relationships, rather than on nouns and things…. Early childhood teachers of Mi'kmaw students would find the use of events as relational concepts from which experience is derived a more useful tool than the use of nouns and categories.* (p. 90)

Lunney Borden (2011) talks about using a verb-based discourse pattern in lessons with Mi'kmaw children, for example focusing on the active properties of geometric objects, rather than identifying and naming their parts. This does not necessarily mean using the first language of the students, but of responding to some of the ways meaning is made in their languages. Working with both Mi'kmaw children and their teachers, Lunney Borden talks about teaching Mi'kmaw children in English, but targeting her English so she used more verbs to describe things and processes and fewer nouns.

CONCEPTS AND CONCEPTUALISATION

The linguistic process of nominalisation, a key feature of the English mathematics register, can be linked to the role of abstraction in concept development. Abstraction is often talked about in mathematics education without being clearly defined. One sort of abstraction can be seen as relating to the degree of connection to the real world, opposed to concreteness; another, empirical abstraction, is used in the development of even concrete concepts (Mitchelmore & White, 2004). Skemp (1987) described this type of abstracting as an activity, a process of becoming aware of similarities between a series of experiences. He called the end product an *abstraction* or *concept*: "Some kind of lasting change, the result of abstracting, which enables us to recognise new experiences as having the similarities of an already formed class" (p. 21). For example, multiple experiences of a single chair and of other chairs eventually enable the creation of a concept "chair". Skemp points out the roles that naming a concept plays in concept development, by helping us to group experiences, helping us learn more complex concepts that are the result of extended cultural and scientific effort, such as acceleration or gravitation, and helping exclude things from a concept.

From this perspective, abstraction and the development of concepts in Indo-European languages is associated with nominalisation. Abstract concepts are nouns. For example, being able "to add" does not include all that we mean by "addition". The process of abstracting a concept parallels the development of an abstract noun from a verb. The very concept of a concept in English is linked to nominalisation. This nominalising process, and naming of concepts, can lead to the attribution of reality to our concepts. While chairs clearly exist, and many people who share similar concepts of chair, the exact conceptual boundaries might differ, such that while one person might easily recognise an upturned crate as a chair, another might not. However, it might fruitful to look more closely at verbs as describing concepts, although English's association of abstraction with nominalisation makes it challenging to talk about abstracting mathematical processes without nominalising them.

These considerations are important in multilingual mathematics education environments. Similar, related concepts in different languages are shaped by cultural, experiential and linguistic factors. Confusion can result when differences between concepts are not recognised. In Malawi, how Chichewa speaking students understand English probability words is founded on probability concepts in Chichewa: "Even though students might use the same words as the teacher, the meanings for students might be very different from that of the teacher" (Kazima, 2007, p. 188). There is a need to explore how mathematical concepts and conceptualisation vary between languages.

FRAMING PATH AND MANNER: VERB-FRAMED, SATELLITE-FRAMED AND SERIAL VERBS

Describing motion, languages differ in what they put together at both word level and sentence level. Talmy's (1985) distinguishes between *verb-framed* and *satellite-framed* languages. Verb-framed languages present the path of motion on the verb, with manner as a subordinate addition, as in the Spanish *La botella entró a la cueva (flotando)* "The bottle moved-in to the cave (floating)". In satellite-framed languages such as English, the manner is contained in the main verb, as in *The bottle floated into the cave*, where the preposition *into* indicates the path (examples from Talmy, 1985, p. 69). Note that verb-framed refers to this specific way that languages package manner, whereas verb-based refers to a language's tendency to encode as verbs more things that are encoded by other languages as nouns.

Cognitive effects of this distinction probably involve the differences between languages in which parts of an expression are compulsory to make a full utterance and which are optional. Slobin (2006) found differences in the mental imagery of Spanish speakers from English speakers related to this distinction: English speakers reported more mental imagery for manner of movement than

Spanish speakers when reading texts that had few manner verbs. Similarly, an investigation of language and event perception tracking eye movements of video scenes with Greek and English speakers found while the language of the speaker did not affect event perception, it affected how people prepared to speak about or to memorise events with respect to differences paid to manner and path of events (Papafragou, Hulbert, & Trueswell, 2008).

Another type of language has what are called *serial verb* constructions, in which both path and manner occur in verbs that are not in dependent relations with each other, used sequentially (Slobin, 2006). Serial verb constructions occur in Ewe, spoken in Ghana (Ameka & Essegbey, 2013) and Kilavila, spoken in Papua New Guinea (Senft, 2006). Serial verbs are common in Iwaidja, and the actions can be sequential or simultaneous (Pym, 1985). It is likely that there are cognitive differences between speakers of serial verb languages and other languages, just as there are differences in mental imagery and memory between speakers of verb-framed and satellite-framed languages.

CONTEXT: IWAIDJA AND MINJILANG

Iwaidja is an Australian language with approximately 140 speakers which is spoken predominantly at Minjilang, an indigenous community in Australia's Northern Territory. Minjilang is a multilingual community with the most widely spoken languages being Iwaidja, Mawng, Kunwinjku and English. Iwaidja and Mawng are in the same language family but historically not the closest linguistic relatives, whereas Kunwinjku is only very distantly related to the other two. In this community, as with much of this region of Australia, multilingualism is a facet of local culture and traditions (Evans, 2010). To be multilingual is the norm and people often add more languages to their linguistic repertoire throughout their lives. English is spoken throughout Minjilang community in places such as the school, the clinic and the local government office, but in all these places local languages are also widely spoken.

At Minjilang, each school class has an English-speaking non-indigenous teacher and a local indigenous assistant teacher who speaks English and one or more of the local languages. As well as assisting with general classroom activities, the indigenous assistant teacher provides local cultural expertise and sometimes translates between the non-local teacher and the students, that is, between English and local languages. Teachers in this community realise that not sharing a common language fluently with their students to be one of the most significant factors affecting their mathematics classrooms, but have little appreciation of conceptual and grammatical differences between the languages (Edmonds-Wathen, 2015).

In terms of a relationship between language and place, Iwaidja is considered to be the indigenous language of Minjilang and is the main language of the

Traditional Owners. However, Kunwinjku and Mawng are actually more widely spoken than Iwaidja at Minjilang, particularly in the younger age groups, and Iwaidja is seriously endangered. The younger generation of adults are reported by the senior adults to be no longer speaking "proper" Iwaidja. Some children can understand Iwaidja but not speak it.

It is likely that the home languages of these students influence their use and conception of spatial language in English. Describing static situations, some Minjilang children use spatial frames of reference in English in ways that resemble their elders' uses of spatial frames of reference in Iwaidja, using the intrinsic frame of reference for terms such as "in front" and "behind" in contexts where speakers of Standard Australian English use the relative frame of reference, that is, using "in front" and "behind" with respect to scene-internal features rather than with respect to the point of view and body of the speaker (Edmonds-Wathen, 2014). There are also similarities in uses of intrinsic spatial frames of reference between Iwaidja, Mawng and Kunwinjku, as well as similarities in use of the absolute frame of reference, that is, using geo-locational reference such as north, south and so on (Edmonds-Wathen, 2013). Investigating mathematical features of Iwaidja therefore provides insights into mathematical ways of thinking and speaking that are likely shared by speakers of nearby languages such as Mawng and Kunwinjku.

THE MAP TASK

This paper reports on some of the results from a map task which I designed to elicit route descriptions. It was a barrier task for two participants, derived from the Human Communication Research Centre (HCRC) Map Task (Anderson et al., 1991). One participant was assigned the role of Director and given a map with a route marked. The other participant was assigned the role of Matcher and given the same or similar map without the route. The Director described the route to the Matcher, who drew the route on their map. The maps were designed to contain only items that would be familiar to Minjilang residents, including having a coastline and landscape features such as a creek and beaches. The items on the maps were shown as drawings rather than symbols, as some of the participants were not familiar with map conventions, and no words were written on the maps. An example of a Director's map is shown in Figure 1.
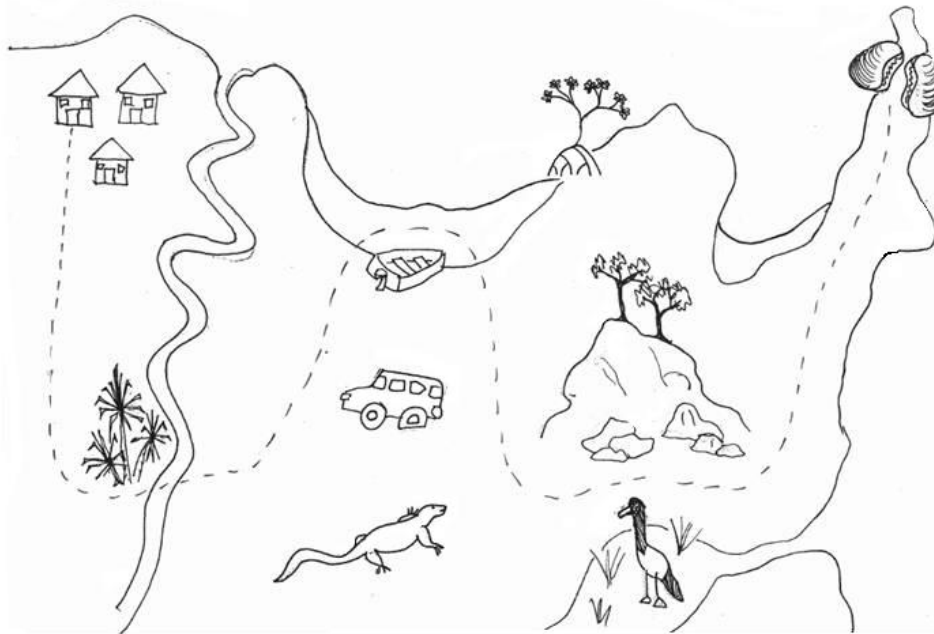
Figure 1. Example of a director's map showing route

This task was part of a linguistic field study documenting Iwaidja spatial language that was embedded in a larger ethnographic project aimed towards developing culturally responsive mathematics education at Minjilang (Edmonds-Wathen, 2013). The participants were six Iwaidja language consultants, senior Iwaidja speakers who were all experienced with participating in language documentation activities. Their English proficiency and levels of formal schooling varied. The study was conducted according to Australian university human research ethics procedures. Participants were provided with a plain language statement that was explained verbally in simple English and in Iwaidja, with the assistance of Bruce Birch, a linguist who speaks Iwaidja. Participants gave written and verbal consent, and chose whether their names would be used in publications. One participant chose to remain anonymous. Speakers' initials are shown with each example, and speakers' names are in Table 1 below.

Table 1
Iwaidja Consultants

Name	Initials	Age in 2010	Sex
Khaki Marrala	KM	82	M
Charlie Mangulda	CM	75	M
Rae Girribuk (deceased)	RG	73	F
Anonymous 1	A1	65-70 (approx.)	M

Table 1
Iwaidja Consultants

Name	Initials	Age in 2010	Sex
David Galak	DG	55-65 (approx.)	M
Maggie Marburrunbi	MM	50-60 (approx.)	F

The task was conducted six times with three pairs of participants who each had a turn at directing and matching. The sessions were videoed with an external microphone. The first pair used matching maps where the only difference was that one had a route marked and the other did not. The other two pairs were given different maps, so that some of the items on the Director's map either did not appear on the Matcher's or were in different locations. The intention was that more complex language would be generated as the participants encountered the differences in their maps. The task was explained to the participants in both English and Iwaidja. Participants in the sessions where the maps differed were informed that the maps might be different. The task allowed participants to choose what types of spatial language they used.

The task elicited route descriptions using motion verbs. Some features of Iwaidja that were used in the task are the serial verb construction, the directional distinction, specific motion verbs to describe ways of passing and circling, sequencing and vertical movement which are presented in more detail below. These route descriptions combine or separate mathematically meaningful components differently to English.

Iwaidja is a verb-based language. Many aspects of life are described as processes rather than things. Kin relations, for example, may be expressed using inflected verbs, so that one says *she sisters me* or *he uncles you* (Evans & Birch, 2007). Verbs often take coverbs or adverbs, many adjectives inflect for number, and it is not always immediately clear whether words are verbs, adjectives or adverbs. Iwaidja is a head-marki ng language, in which affixes, both prefixes and suffixes, provide information including subject, object, direction and tense with regard to a stem.

In general, spatial verbs consist of a stem with a prefix indicating either the subject (in the case of intransitive verbs) or both subject and object (in the case of transitive verbs). Past tense is indicated by suffixes. Future tense is indicated via a separate prefix. In addition, there is an optional prefix indicating directionality, either AWAY from a deictic centre or TOWARDS it (glossed TO). Gloss abbreviations are given at the end of the paper in Table 2 below. Without this prefix the directionality can be considered neutral. These directions are with respect to the deictic location, which may or may not be the speaker's location. This prefix sometimes combines with subject and object prefixes as well as the

future tense prefix to form a single morpheme (Pym & Larrimore, 1979). Examples include *jan-ara,* glossed AWAY.I.FUT-go, "I will go", and *nyan-ara*, glossed TO.I.FUT-go, "I will come". An example of a prefix which combines subject and object is *r-*, glossed he.to.it, which means "he (third person singular masculine subject) acts upon it (third person singular object)".

Table 2
Abbreviations

Abbreviation	Meaning	Abbreviation	Meaning
Ø	empty morpheme	LOC	locative
AWAY	away	NPST	non past
DEM	demonstrative	PAST	past
FUT	future	REL	relative particle
ITER	iterative	TO	towards

Examples in Iwaidja are shown in four lines. The first line in bold shows the Iwaidja sentence or phrase in the everyday orthography. The second line in italics shows each word. The Iwaidja words have hyphens separating morphemes (meaningful word parts). The third line shows an aligned morpheme by morpheme English gloss. Where the Iwaidja word is translated by more than one English word, but these cannot be separated morphemically in the original, the English words are separated by a period. For example, *artirran* "he came back" can be separated into *art-* "towards; he/she/it", *irra* "come back" and *–n*, which marks the past tense. The third person singular pronoun and the towards direction marker cannot be separated in the morpheme *art-*, nor can the "come" and "back" in *irra*. Hence, *art-irra-n* is glossed "TO.he-come.back-PST". The final line shows a free English translation of the sentence or phrase.

All transcriptions and translations were done with the assistance of a fluent native speaker, Joy Malwagag Williams (deceased). Joy was one of the most experienced Iwaidja translators who had also worked as an assistant teacher at the school in Minjilang for many years. In some cases, translations required extended discussion including testing alternative uses of the words. Other sources on Iwaidja including Pym (1985), Pym and Larrimore (1979), and Bruce Birch's working dictionary were consulted as necessary.

IWAIDJA ROUTE DESCRIPTIONS

The predominate approach of the six participants was to use specific verbs of motion. Example 1 illustrates many typical characteristics of Iwaidja motion description. This example, just like others in text, entails three sections: original text, translation between words, and final translation text.

Example 1. Artirran ararlarrngbung abulakuny awaran. Yabulakuny wardad ba ajbud. Yartirran yarnukbun jumung murrhala ari. Yartirran yarnukbun wuka jumung narrhardi bani. Yartirran barakbarda yariman ba alan ba yawaran, barakbarda jumung birtbani, arrarnarn lda arrayi (A1).

Translation between words

art-irra-n	*a-rarlarrngbu-ng*	*a-bulaku-ny*
TO.he-come.back-PST	TO.he.to.it-turn-PST	TO.he-come.down-PST

aw-ara-n	*ya-bulaku-ny*		*wardad*	*ba*
TO.he-come-PST	AWAY.he-come.down-PST		one	the

ajbud	*yart-irra-n*	*y-arnukbun*	*jumung*
beach	AWAY.he-come.back-PST	AWAY.he-turn.off	REL

murrhala	*ari*	*yart-irra-n*		*y-arnukbun*
pandanus	it.stand	AWAY.he-come.back-PST		AWAY.he-turn.off

wuka	*jumung*	*narrhardi*	*bani*	*yart-irra-n*
LOC	REL	goose	it.sits	AWAY.he-come.back-PST

barakbarda	*yari-ma-n*		*ba*	*alan*	*ba*
that.one	AWAY.he.to.it-take-PST		the	track	the

yaw-ara-n	*barakbarda*	*jumung*	*birt-bani*	*arrarnarn*
AWAY.he-come-PST	that.one	REL	they-sit	milky.oyster

l*da*	*arrayi*
and	black.lip.oyster

Final translation text. "He came back, he turned, he came down, he came. He went down to one beach. He turned back to where the pandanus tree is. He turned back to where the goose was sitting. He went back there and he took that road, he went along where that thing is, milky oysters and black lip oysters."

This Iwaidja example has more motion verbs than the English translation including a first sentence which consists of just four verbs in a serial verb construction and has various uses of the directional prefixes. Some of the directional contrasts and path details of the verbs are represented in the English translations with prepositions such as *back* and *down*.

Directional Distinction
This example includes several uses of the TOWARDS and AWAY prefixes. The basic movement verb *ara* which means "go" or "come" depending on the context is seen in the past third person singular forms *awaran* "he came" and *yawaran* "he went". We also see the verb *irra* which means "go back" or "come back" depending on the direction, in the forms *artirran* "he came back" and *yartirran* "he went back". *Irra* thus refers to a change of direction. The example also

contains both TOWARDS and AWAY forms of *wulaku* "go down", in *abulakuny* "he came down" and *yabulakuny* "he went down". Hence the use of the directional contrast enriches the semantic scope of the verbs to which it is applied.

Serial Verb Construction and Event Conception

We can also see the serial verb construction which occurs frequently in Iwaidja. The first sentence consists only of four verbs, all taking the TOWARDS prefix. *Artirran ararlarrngbung abulakuny awaran* "He came back, he turned, he came down, he came". A colloquial English translation might be something along the lines of "he turned and came straight back down". The use of a series of verbs in this context does not necessarily represent a series of actions that follow each other. Rather, the imagined route follower's single, if complex, act of turning, returning and descending are conceptually packaged together despite each being expressed as independent verbs which could each stand as a sentence themselves in Iwaidja. Foley (2004) describes a similar expression in the Yimas language of Papua New Guinea as a *macro-event*.

> *The event of coming back to a place does possess the inherent component sub-events of turning around, retracing one's path and arriving at that place. Yimas…seems excessively concerned with spelling out these inherent components, while English is happy with just expressing the directionality of motion, "come", and the path, "back".*
> (p. 133)

In any language, you would expect the language of motion to involve the use of verbs. However, in verb-framed languages, manner of motion is expressed outside the verb, for example as an adverb. In satellite-framed languages the path is expressed outside the verb, for example with a prepositional phrase. Iwaidja's serial verb constructions frequently combine both manner and path in a series of verbs, as in Example 1. The serial verb construction in Iwaidja suggests that speakers pay attention to sub-events of complex motions.

Ways of Passing

Iwaidja encodes specific spatial information into distinct, although related verbs. The range of verbs elicited to refer to passing objects on the map is a good example of this. The most general of these is *marraywung* "pass". It is a transitive verb, requiring the specification of what is passed, but does not provide any more detailed information about how the object is passed. Its use is shown in Example 2.

Example 2. Rimarraywung yawarang mangawala ajbud jumung kabala ari (CM).

Translation between words

ri-marraywung	*yaw-ara-ng*	*mangawala*	*ajbud*	*jumung*
he.to.it.pass	AWAY.he-go-NPST	fast	beach	REL

kabala	*ari*
boat	it.stand

Final translation text. "He goes past it and runs along the beach where boat is."

There are also verbs derived from *marraywung* for passing in front, behind or to the side of objects. Their use depends upon those objects having an intrinsic front, back or side. The verbs are *marlmarraywung* "pass behind", Example 3; *wudbarraywung* "pass in front", Example 4; and *ngunyunmarraywung* "pass beside", Example 5.

Example 3. Artirran ararnukbung rimarlmarraywung wuka jumung mudika wulurr (CM).

Translation between words

art-irra-n	*ar-arnukbu-ng*	*ri-marlmarraywung*
TO.he-come.back-PST	TO.he-turn.off-PST	he.to.it-pass.behind

wuka	*jumung*	*mudika*	*wulurr*
LOC	REL	car	back

Final translation text. *"*He came back, turned off and passed behind the back of the car."

Example 4. Kirrimul warrkarrk aju riwudbarraywung (CM).

Translation between words

kirrimul	*warrkarrk*	*aju*	*ri-wudbarraywung*
like	goanna	it.lie	he.to.it-pass.in.front

Final translation text. "He passes in front of where the goanna is."

Example 5. Kabanangunyunmarraywun baraka dinghy (KM).

Translation between words

kabana-ngunyunmarraywu-n	*baraka*	*dinghy*
you.to.it.FUT-pass.beside-NPST	DEM	dinghy

Final translation text. "You will pass beside the dinghy."

Ngunyunmarraywung "pass beside" contains the body part stem *ngunyun* "waist" which also occurs in words such as *mangunyuni* "beside" and *angunyulunmin*

"side by side". The derivational origins of *marlmarraywung* "pass behind" and *wudbarraywung* "pass in front" are not so transparent. They do not have any relation to the words *warrwak* "behind" and *wurdaka* "in front". The examples here of the various verbs of passing indicate more than just choices in the expression of "beside", "behind" and "in front". Rather, there are differences in their expression depending on whether one is talking about static location or about motion.

Sequencing and Ordinality

Warrwak "behind" (an adverb) and *wurdaka* "in front" (a verb) are common words and were used by the Iwaidja speakers frequently in activities involving description of static location (Edmonds-Wathen, 2014). In the Map task they were used in the senses of "first" and "last". In Example 6, *wurdaka* "first" was used to describe the origin of the road. The verb *ang...ldakbanakbi* "last" was also used.

Example 6. Wurdaka abalkbang burdan ngarrkarrk lda wardad kurrambalk, wardad bani angbuldakbanakbi (CM).

Translation between words

w-urdaka	*a-balkba-ng*		*burdan*	*ngarrkarrk*	*lda*
it-first	To.it-come.out-PST		from	two	and

wardad	*kurrambalk*		*wardad*	*bani*	*angbu-ldakbanakbi*
one	house		one	it.sit	it.to.it-last

 Final translation text. "It starts from where those three houses are, (from the) one that's sitting last."

While *warrwak* "behind, last" is an adverb and does not inflect, *ang...ldakbanakbi* "last" is a transitive verb such that the subject "lasts" or "comes last to" the object.

A dynamic, verb-based worldview affects how ordinality is perceived. Objects or people in a line, in an order, do not just occupy their places, they follow each other. In English, a number that comes "after" is "higher" or "greater", and a number that comes "before" is "lower" or "smaller". In Iwaidja, the concepts of "first", "in front", "go before" are combined in one word, *wurdaka*, while "behind", "later" and "after" are combined in *warrwak*.

Ordinality may also be seen as a transitive relation. At Minjilang, I have heard *second* being used as a transitive verb by students getting into a line, as in "You second her and I'll second you," where *second* means "follow" or "come immediately after, come next". Rather than naming a place, it describes a relation between two bodies. The transitive verb *ang...ldakbanakbi* "last" was initially translated by Joy both as "second" or "third", according to the context of it referring to the last of two or three objects. Bruce suggested that this was

surprising and unlikely to be accurate, so the three of us discussed the word and how it could be used, until we could come to a more accurate translation. Some of the difficulty in eliciting a more accurate translation of this term may have been due to its transitivity, the fact that it says that the subject "lasts" or "comes last to" the object, which is grammatically different from English *last*.

Circling

Another set of verbs which are potentially of mathematical interest are those which were used in the Map task for "go around" or "circle". There were four of these, with the simplest *alnga* appearing as a stem in the other three. Examples are given from the Map task of *alnga*, Example 7, *ngardalnga*, Example 8, *ldakburdalnga*, Example 9, and *marralnga*, Example 10.

Example 7. Abanalnga kabala nga? (RG). Kabanalnga ba kabala yukbanirra yukbanawulakun (MM).

Translation between words

aban-alnga		*kabala*	*nga*
To.I.to.it.FUT-go.around		boat	eh

kaban-alnga	*ba*	*kabala*	*yukban-irra*
you.to.it-go.around	the	boat	TO.you.FUT-come.back

yukbana-wulaku-n
TO.you.FUT-come.down-NPST

Final translation text. RG: "I go around the boat, don't I?".MM: "You go around the boat and then you come back down."

Example 8. Kabanayan mana ajbud aju. Kabanangardalnga barakbarda yukbanawulakun (MM).

Translation between words

kaban-ayan		*mana*	*ajbud*	*aju*	*kabana-ngardalnga*
you.to.it.FUT-see-NPST		maybe	sand	it.lie	you.to.it.FUT-go.around

barakbarda	*yukbani-wulaku-n*
there	to.you.FUT-come.down-NPST

Final translation text. "You see the sand there. Go around that and come down."

Example 9. Barakbarda rtuwa rildakburdalngan. Awaran mangawala j…. ari abalkbang manyij wakaldakan. Anbirrk mardan yawaran rildakburdalngan wuka (DG).

Translation between words

barakbarda	*rtuwa*	*ri-ldakburdalnga-n*	*aw-ara-n*
there	just	he.to.it-go.around-PST	TO.it-go-PST

mangawala	*ari*	*abalkbang manyij*	*w-akaldakan*	*anbirrk*
fast	it.stand	east	it-on.side	hill

mardan	*yaw-ara-n*	*ri-ldakburdalnga-n*	*wuka*
little	AWAY.it-go-PST	he.to.it-go.around-PST	LOC

Final translation text. "It just went around that [pandanus]. It came straight to where...., it's on the east side. It went around the little hill there."

Example 10. Nganamarralngaka (RG).

Translation between words

ngana-marralnga-ka
I.FUT-circle-ITER

Final translation text. "I'll circle (it)."

Marralnga has been glossed as "(to) circle" while *alnga, ngardalnga* and *ldakburdalnga* have been glossed as "go around". However, the different scopes of each term are not clear. From the uses in the Map task and discussion with Joy, the semantic difference between the terms is not one of scale (large circles versus small circles) nor one of completion. *Alnga, ngardalnga* and *ldakburdalnga* are all transitive, requiring an object to be circled, whereas *marralnga* has intransitive morphology. However, in Example 10 (RG) used an iterative form of *marralnga* while she drew a circle right around the boat. Like *marlmarraywung* "pass behind" and *wudbarraywung* "pass in front", the derivational origins of *ngardalnga, ldakburdalnga* and *marralnga* are not transparent. Discussing these terms, Joy said that there was no noun for a circle in Iwaidja. The attributes of action are favoured over the naming of shape, and geometric concepts are more likely to be expressed through verbs, as actions.

Verticality
While Iwaidja has adverbs *yurrngud* "on top, above" and *wuwud* "below", used to describe static location, the verbs *bulaku* "go down, descend" and *wurti* "go up, ascend" were used in the Map task to describe motion up or down. Normally in Iwaidja the seaward direction is "down" and the inland direction is "up". However, in this task, participants also used the top and bottom of the map as directions. In Example 11, DG talks about the track going "up" in a direction towards the coast, because it is going "up" the map, and in Example 12, MM uses *wulaku* "go down" for a movement away from the coast, as the track is going "down" the map.

Example 11. Yartirran yawurtin ngalaj ari balarra ba narrhardi ari bani wakaldakan wuka (DG).

Translation between words

yart-irra-n	*ya-wurti-n*	*ngalaj*	*ari*
AWAY.it-go.back-PST	AWAY.it-go.up-PST	LOC	it.stand

ø-balarra ba	*narrhardi*	*ari*	*bani*
it-middle the	magpie.goose	it.stand	it.sit

w-akaldakan	*wuka*
it.on.side	LOC

Final translation text. "It went back up in the middle where the magpie goose is sitting on the side."

Example 12. Janawulakun jumung mudika ari janara angmanarrajbang wuka jumung ingbal (MM).

Translation between words

jana-wulaku-n	*jumung*	*mudika*	*ari*
AWAY.I.imp-go.down-NPST	REL	car	it.stand

jan-ara	*angmana-rrajbang*	*wuka*	*jumung*	*ingbal*
AWAY.I.fut-go	TO.I.FUT-walk	LOC	REL	reef

Final translation text. "I'll go down where the truck is and then I'll walk to the reef."

It is far from certain to what extent speakers of Iwaidja link the location of *wuwud* "below" with the motion of *bulaku* "go down, descend", or the location of *yurrngud* "on top, above" with the motion of *wurti* "go up, ascend", and further investigations would be needed to investigate this.

CONCEPTUALISATION

Route descriptions in Iwaidja show attention to sub-events of complex motions, attention to both path and manner in main verbs and groups of specific verbs that combine both path and manner. The "behind-ness" and "in front-ness" conceptualised in the *marlmarraywung* "pass behind" and *wudbarraywung* "pass in front", is not lexically conflated with that in *warrwak* "behind" and *wurdaka* "in front". Satellite-framed English has the same way of specifying location and path in similar expressions. In English, the grammar and concept of "down-ness" applies to both "being down" and "going down". Iwaidja groups its "passing" verbs and its "circling" verbs, as seen from common roots in each group. However, the lexicalisation of the specific verbs means that a speaker may not be conscious of the grouping to the same extent as if a common word was used. To

go and to come, to move towards or away from the speaker or other deictic centre, are also distinguished on the verb in Iwaidja.

While this data was collected from older speakers, it is likely that some of these ways of conceptualising are shared by the children of Minjilang, whether or not they speak Iwaidja. Some of these children's conceptions of static location parallel the Iwaidja ones, such as their use of the intrinsic sense of "in front" and "behind" in contexts where the relative sense is generally used in English (Edmonds-Wathen, 2014). The use of terms like "second" as a transitive verb also parallels Iwaidja. Influences of indigenous Australian languages on the spatial language of Kriol can also be shown. Kriol, a creole language with an English-based lexicon spoken by many indigenous Australian children as their first language (although not much spoken at Minjilang), also expresses path in basic motion verbs. Examples include *guwap* "go up, ascend" and *gudan* "go down, descend". Instead of the spatial prepositions that are so widely used in mathematics in English, Australian indigenous languages including Iwaidja, Mawng and Kunwinjku have a tendency to describe path on the verb, resulting in differently specific spatial verbs to English.

IMPLICATIONS FOR MATHEMATICS EDUCATION

As well as being taught mathematics in English by English-speaking teachers, most Australian indigenous language speaking children are assessed in English. When English-speaking teachers talk and think about how to teach the language of mathematics, they tend to package concepts such as spatial concepts in the way that English packages them. Meaney and Evans (2013) discuss the conceptual expectations of English speaking teachers of indigenous language speaking students with respect to the ordinality and cardinality that is combined in English numbers. They point out that numbers in some Australian languages have only cardinality, and that this may be one factor that affects the difficulty some indigenous students have learning to use terms like "before" and "after" with respect to numbers.

> If a teacher expects a number label to include an understanding of ordinality, then when children are unable to discuss numbers that come before or after, using this ordinality understanding, then it is often the child who is considered to have the problem. Instead, it would be better to recognise that the issue lies with the teacher misunderstanding how the counting words are used by the children. (p. 487)

As we have seen, in Iwaidja, ordinality appears as a transitive relation, which is something English-speaking teachers may not be familiar with, and which may not be considered in school curricula.

Teachers cannot always investigate the mathematical concepts and grammatical features of their students' languages themselves. Hence teachers of

students who speak languages other than the language of instruction need mathematics teacher training which includes learning about mathematically significant grammatical structures. Teachers may view mathematical concepts, especially those they regard as basic, as having an essential, if non-concrete, reality. One of the biggest challenges for teachers is to realise that their own basic interpretations of material reality are linguistically and culturally mediated and may not be shared by their students.

The verb-based nature of Australian languages has not been highlighted in the mathematics education literature. Educationally, one possible strategy would be for teachers to use elements of the grammatical patterns of their indigenous language speaking students in teaching the language of location and motion in English (Lunney Borden, 2011). Activities for teaching route descriptions in English to indigenous language speaking students could include, in appropriate order, teachers describing a route through an obstacle course and children following it, children describing a route they have just followed, children giving each other instructions to follow routes, children drawing and describing routes, and eventually children using similar route description barrier tasks to that used in this study. In English it is usual to leave out a movement verb in descriptions of a list of movements, as in "go around the chair, under the table and over the mat". With children who speak verb-based languages, although it may seem counter-intuitive and unnatural to native English speaking teachers, whole verb phrases should be used each time, as in "go around the chair, go under the table and go over the mat". While the spatial prepositions may be emphasised in describing static location, when describing motion the path should not be separated or emphasised: Rather than "down, down, go down", the whole verb phrase "go down, go down, go down" could be repeated. Of course, the students eventually need to learn standard English grammatical forms, but that could be the focus of later lessons, after the conceptual learning of these early lessons. Early years mathematics lessons on shapes could describe a circle by its capability of rolling, or a square by its capability of sitting, in advance of focusing on the number of corners or edges.

The examples in this paper are far from exhaustive in describing the differences between the language of route description in Iwaidja and in English. Some general tendencies have been identified, such as Iwaidja's packaging of both path and manner into serial verb constructions and the implications of this for the formation of some mathematically significant spatial concepts. This paper has aimed to broaden the understanding within mathematics education and mathematics education research of the possible range of conceptual variety in different languages, focussing on grammatical structures. Understanding these grammatical structures can help mathematics education researchers to design investigations of conceptual and cognitive differences between speakers of different languages. For example, the cognitive effects of using serial verb

constructions could be investigated to add to comparisons between path-framedand satellite-framed languages.

These conceptual differences have cognitive implications which are relevant to the design of mathematics word questions in different languages. The grammatical structures of mathematics questions, which are used in classroom exercises and tests, serve to focus attention on different parts of the information in the questions. Differences in grammatical structure between languages may result in a question that focuses attention on one part of the information in one language and in a question that is intended to be equivalent in another language focusing attention on one part of the information. This could potentially promote different problem solving strategies. More research is required to understand the relationships between grammatical structure and problem solving strategies in different languages.

Research into mathematical concepts in indigenous languages and the cognitive implications for speakers of variety in those concepts needs to be done collaboratively with the indigenous communities that it involves. Languages embody worldviews including intangible intellectual property. As others including Pinxten et al. (1983) and Meaney and Evans (2013) have pointed out, languages must not simply be co-opted for the purposes of Western mathematics education. Investigations into the grammar of basic mathematical concepts provide us with the opportunity to look more deeply into our assumptions about these concepts. This in turn may inspire researchers and educators to think creatively beyond the grammatical constraints of their own languages.

REFERENCES

Ameka, F. K., & Essegbey, J. (2013). Serialising languages: Satellite-framed, verb-framed or neither. *Ghana Journal of Linguistics, 2*(1), 19-38.

Anderson, A. H., Bader, M., Bard, E. G., Boyle, E., Doherty, G., Garrod, S.,… Weinert, R. (1991). The Hcrc map task corpus. *Language and Speech, 34,* 351-366.

Barton, B. (2009). *The language of mathematics: Telling mathematical tales.* New York, NY: Springer.

Battiste, M. A., & Henderson, J. S. Y. (2000). *Protecting indigenous knowledge and heritage: A global challenge.* Saskatoon, Canada: Purich.

Edmonds-Wathen, C. (2013). *Frame of reference in Iwaidja: Towards a culturally responsive early years mathematics program.* (Doctoral dissertation, RMIT University, Melbourne, Australia). Retrieved from https://researchbank.rmit.edu.au/view/rmit:160446

Edmonds-Wathen, C. (2014). Influences of indigenous language on spatial frames of reference in Aboriginal English. *Mathematics Education Research Journal, 26*(2), 169-192.

Edmonds-Wathen, C. (2015). Indigenous language speaking students learning mathematics in English: Expectations of and for teachers. *Australian Journal of Indigenous Education, 44*(1), 48-58.

Evans, N. (2010). *Dying words: Endangered languages and what they have to tell us*. Chichester, United Kingdom: Wiley-Blackwell**.**

Evans, N., & Birch, B. (2007, June). *Being so fathered and so husbanded: Kinship verbs in Iwaidja.* Paper presented at the DoBeS Workshop, MPI Nijmegen, The Netherlands.

Foley, W. A. (2004). The notion of "event" and serial verb constructions: Arguments from New Guinea. In W. Khanittanan & P. Sidwell (Eds.), *SEALS XIV: Papers from the 14th annual meeting of the Southeast Asian Linguistics Society* (pp. 129-156). Canberra, Australia: Pacific Linguistics.

Halliday, M. A. K. (1978). Sociolinguistic aspects of mathematical education. In *Language as social semiotic: The social interpretation of language and meaning* (pp. 194-204). London, United Kingdom: Edward Arnold.

Kazima, M. (2007). Malawian students' meanings for probability vocabulary. *Educational Studies in Mathematics, 64*(2), 169-189.

Lunney Borden, L. (2011). The "verbification" of mathematics: Using the grammatical structures of Mi'kmaq to support student learning. *For the Learning of Mathematics, 31*(3), 8-13.

Lunney Borden, L. (2012). What's the word for…? Is there a word for…? How understanding Mi'kmaw language can help support Mi'kmaw learners in mathematics. *Mathematics Education Research Journal, 25*(1), 5-22.

Meaney, T., & Evans, D. (2013). What is the responsibility of mathematics education to the indigenous students that it serves? *Educational Studies in Mathematics, 82*(3), 481-496.

Mitchelmore, M., & White, P. (2004). Abstraction in mathematics and mathematics learning. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 329-336). Bergen, Norway: PME.

Papafragou, A., Hulbert, J., & Trueswell, J. (2008). Does language guide event perception? Evidence from eye movements. *Cognition, 108*(1), 155-184.

Phakeng, M. S., & Moschkovich, J. N. (2013). Mathematics education and language diversity: A dialogue across settings. *Journal for Research in Mathematics Education, 44(*1), 119-128.

Pinxten, R., van Dooren, I., & Harvey, F. (1983). *Anthropology of space: Explorations into the natural philosophy and semantics of the Navajo.* Philadelphia, PA: University of Pennsylvania Press.

Pym, N. (1985). Iwaidja verbal clauses. In S. K. Ray (Ed.), *Work papers of SIL–AAB* (Series A, Volume 9, pp. 39-52). Darwin, Australia: Summer Institute of Linguistics.

Pym, N., & Larrimore, B. (1979). *Papers on Iwaidja phonology and grammar.* Darwin, Australia: Summer Institute of Linguistics.

Senft, G. (2006). Prolegomena to a Kilivila grammar of space. In S. C. Levinson & D. Wilkins (Eds.), *Grammars of space: Explorations in cognitive diversity* (pp. 206-229). Cambridge, United Kingdom: Cambridge University Press.

Skemp, R. R. (1987). *Psychology of learning mathematics.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Slobin, D. (2006). What makes manner of motion salient? Explorations in linguistic typology, discourse and cognition. In M. Hickmann & S. Robert (Eds.), *Space in languages: Linguistic systems and cognitive categories* (pp. 59-81). Amsterdam, The Netherlands: John Benjamins.

Staats, S. (2009). Somali mathematics terminology: A community exploration of mathematics and culture. In R. Barwell (Ed.), *Multilingualism in mathematics classrooms: Global perspectives (*pp. 32-46). Clevedon, United Kingdom: Multilingual Matters.

Talmy, L. (1985). Lexicalization patterns: Semantic structure in lexical form. In T. Shopen (Ed.), *Language typology and syntactic description* (pp. 57-149). Cambridge, United Kingdom: Cambridge University Press.

Cris Edmonds-Wathen
Umeå University
Charles Darwin University
cris.edmonds-wathen@umu.se